Mental Modeler Matrix Analysis

This document describes the analysis of the weights matrix, according to Özesmi and Özesmi 2004 [Ö2004]. Further details can be found in the cited paper and the Mental Modeler web application manual [MMM].

General Remarks

Let's start with a sample model (fig.0). It consist in a graph of nodes (A, B, …., K) and some connecting arrows, each arrow brings a weight in the range [-1 … +1]. The weight of the connection from the node i to the node j is called w_{ij}. e.g. the weight w_{HF} from node H to node F is positive (blue arrow), while w_{CG} from node C to node G is negative (red arrow)
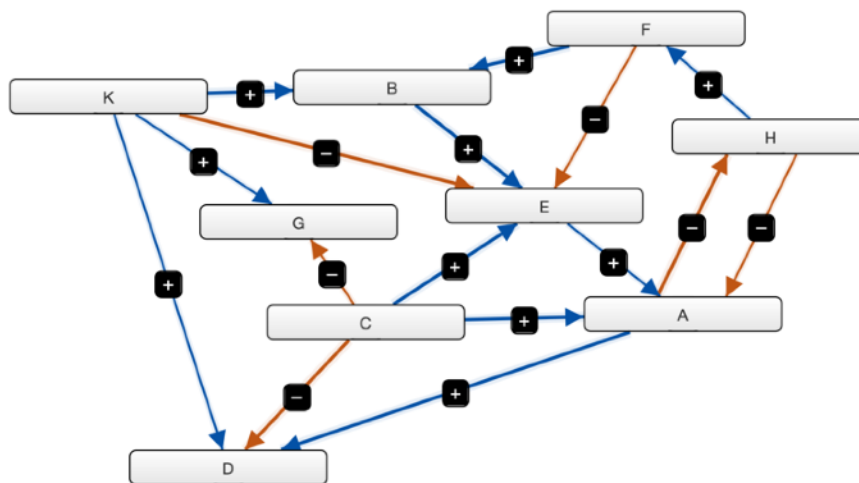

Figure 0 - A sample abstract model.

The rules of the game are the following:

- no self-interaction: i.e. $w_{ii} = 0$ for each node i,
- a positive weight w_{ij} means that the relation from i to j is a direct (proportional) one,
- a negative weight w_{ij} means that the relation from i to j is an inverse (inversely proportional) one,
- $w_{ij} \neq 0$ does not imply $w_{ji} \neq 0$, better yet, criss-crossed relationships are extremely rare, though not impossible, and should be considered with care,
- circular relations (e.g. $H \rightarrow F \rightarrow E \rightarrow A \rightarrow H$) are allowed,
- a node without entering arrows, called a **driver**, acts as a control knob for modeling scenarios,
- a node without exiting arrows, called a **receiver**, is considered an outcome,
- a node with entering and exiting arrows is called an **ordinary** node, it is as much important as the number of connections with the other nodes.

The Mental Modeller web application takes care of all the mentioned rules, through a graphical design interface (fig. 0). The weight matrix can be visualised within the application via the **Matrix** tab (fig.1). The leftmost column bears the names of the arrows' originating nodes, while the top row bears the names of the receiving nodes. we see, e.g., that $w_{FE} = -0.39$, i.e. the arrow from F to E stands for an inverse

relationship: an increase of F causes a decrease of E and vice versa. Pay attention to the direction of the relationship: causes on the leftmost column ad outcomes on the top row.

	A	B	C	D	E	F	G	H	K
A		▾	▾	0.57 ▾	▾	▾	▾	-0.51 ▾	▾
B	▾		▾	▾	0.52 ▾	▾	▾	▾	▾
C	0.66 ▾	▾		-0.55 ▾	0.67 ▾	▾	-0.4 ▾	▾	▾
D	▾	▾	▾		▾	▾	▾	▾	▾
E	0.29 ▾	▾	▾	▾		▾	▾	▾	▾
F	▾	0.65 ▾	▾	▾	-0.39 ▾		▾	▾	▾
G	▾	▾	▾	▾		▾		▾	▾
H	-0.43 ▾	▾	▾	▾	▾	0.33 ▾	▾		▾
K	▾	0.64 ▾	▾	0.26 ▾	-0.63 ▾	▾	0.43 ▾	▾	

Figure 1 - The weight matrix of fig.0 model.

Some details on the nodes can be found in the Preferred State & Metrics tab of the application (fig.2). We will see in the following how to compute them using a spreadsheet.

Total Components
9

Total Connections
16

Density
0.2222222222

Connections per Component
1.7777777778

Number of Driver Components
2

Number of Receiver Components
2

Number of Ordinary Components
5

Complexity Score
1

Component	Indegree	Outdegree	Centrality	Preferred State	Type
A	1.38	1.08	2.46	▾	ordinary
B	1.29	0.52	1.81	▾	ordinary
C	0	2.280000000000002	2.280000000000002	▾	driver
D	1.3800000000000001	0	1.3800000000000001	▾	receiver
E	2.21	0.29	2.5	▾	ordinary
F	0.33	1.04	1.37	▾	ordinary
G	0.8300000000000001	0	0.8300000000000001	▾	receiver
H	0.51	0.76	1.27	▾	ordinary
K	0	1.9600000000000002	1.9600000000000002	▾	driver

Figure 2 - Some node analysis.

In particular, the **indegree** and the **outdegree** of a node are defined as the sum of the absolute values of the weights of the entering and exiting arrows, respectively [Ö2004]:

$$id_i = \sum_j |w_{ji}| \qquad \text{and} \qquad od_i = \sum_j |w_{ij}|$$

pay attention to the order of the indices: w_{ij} goes <u>from</u> node i <u>to</u> node j.

The **centrality** is defined as the sum of in- and outdegree: $cen_i = \sum_j |w_{ji}| + \sum_j |w_{ij}|$.

The mentioned parameters are a relative measure of the importance of a node as a driving one (outdegree), as a receiving one (indegree) and as a joint in the model (centrality). In particular, drivers have a null outdegree, while receivers have a null indegree. Other measures of the importance of a node are the number of entering and exiting connections.

As per the model itself, some structural measures can be evaluated [Ö2004]:

- the **density** d is the ratio of the total number of arrows M with respect to the maximum allowed number of possible connections for N nodes: $d = \dfrac{M}{N(N-1)}$, the d cannot exceed 1 (all nodes connected).

- the **complexity** c of the model is defined as the ratio of the number of receivers over the drivers, note that, despite the name, it does not measure how rich and involved the relationships among nodes are.

- the **hierarchical index** h is defined as

$$h = \frac{12}{(N+1)\,N\,(N+1)} \sum_i \left[\frac{od_i - \left(\sum_j od_j \right)}{N} \right]^2$$

a hierarchical index $h = 0$ corresponds to a *fully democratic* model, while $h = 1$ marks a *fully hierarchical* model, see [Ö2004] for details.

The Mental Modeller Web Application allows to export the weight matrix (i.e. the model itself) as a comma-separated-values (csv) file, that looks like this:

```
"","A","B","C","D","E","F","G","H","K"
"A","","","","0.57","","","","-0.51",""
"B","","","","","0.52","","","",""
"C","0.66","","","-0.55","0.67","","-0.4","",""
"D","","","","","","","","",""
"E","0.29","","","","","","","",""
"F","","0.65","","","-0.39","","","",""
"G","","","","","","","","",""
"H","-0.43","","","","","0.33","","",""
"K","","0.64","","0.26","-0.63","","0.43","",""
```

such model.csv file is the starting point for the weight matrix analysis sketched in the following section.

Practical Spreadsheet Procedure

The first step, obviously, is the import of the csv file into the spreadsheet[1] as follows:

Figure 3 - The freshly imported csv.
Note the same arrow highlighted in fig.1.

The second step consists in adding a few extra columns: five columns for holding the results of the calculation of: the number of entering arrows (**IN**), the number of exiting arrows (**OUT**), the indegree (**ID**), the outdegree (**OD**) and centrality (**CEN**).

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[1] This example has been produced with Apple Numbers.

It is also advisable to mark the diagonal elements, it will be useful for the final polish of the table (fig.4a):

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K
A									0.57				-0.51	
B										0.52				
C						0.66			-0.55	0.67		-0.40		
D														
E						0.29								
F							0.65			-0.39				
G														
H						-0.43					0.33			
K							0.64		0.26	-0.63		0.43		

Figure 4a - Colouring the weights is just for reference.

Now add a row below and as many columns as the nodes number to the right (fig.4b), obtaining an augmented matrix. The augmented elements will contain the absolute values of the weights (fig.5)

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A									0.57				-0.51										
B										0.52													
C						0.66			-0.55	0.67		-0.40											
D																							
E						0.29																	
F							0.65			-0.39													
G																							
H						-0.43					0.33												
K							0.64		0.26	-0.63		0.43											

Figure 4b - The augmented matrix.

The function for the evaluation of the absolute value is generally called ABS(). It is possible -but unlikely- that a localised operative system implies different spreadsheet function names.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A									0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00
B										0.52					0.00	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.00
C						0.66			-0.55	0.67		-0.40			0.66	0.00	0.00	0.55	0.67	0.00	0.40	0.00	0.00
D															0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E						0.29									0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F							0.65			-0.39					0.00	0.65	0.00	0.00	0.39	0.00	0.00	0.00	0.00
G															0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H						-0.43					0.33				0.43	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
K							0.64		0.26	-0.63		0.43			0.00	0.64	0.00	0.26	0.63	0.00	0.43	0.00	0.00

Figure 5 - The absolute values $|w_{ij}|$ in the augmented matrix.

Exploiting the weights w_{ij} and the absolute weights $|w_{ij}|$ of the augmented matrix we can readily evaluate our variables of interest: **OUT** is evaluated as the COUNT() of the weights of the matrix (fig.6a) and **OD** is evaluated as the SUM() of the augmented weights (fig.6b). Repeat for all the rows.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A		2							0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00

Figure 6a - Evaluation of **OUT** as `COUNT(weights)`.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A		2		1.08					0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00

Figure 6b: Evaluation of **OD** as `SUM(absolute weights)`.

Now evaluate the IN and ID parameters applying the `COUNT()` and `SUM()` column-wise, following the respective node names as they appear in the top row of the weights and augmented matrix (fig.7).

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A		2		1.08					0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00
B		1		0.52						0.52					0.00	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.00
C		4		2.28		0.66			-0.55	0.67		-0.40			0.66	0.00	0.00	0.55	0.67	0.00	0.40	0.00	0.00
D		0		0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E		1		0.29		0.29									0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F		2		1.04			0.65			-0.39					0.00	0.65	0.00	0.00	0.39	0.00	0.00	0.00	0.00
G		0		0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H		2		0.76		-0.43				0.33					0.43	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
K		4		1.96			0.64		0.26	-0.63		0.43			0.00	0.64	0.00	0.26	0.63	0.00	0.43	0.00	0.00
						3									1.38								

Figure 7 - Left: **IN** is `COUNT(weights)` - Right: **ID** is `SUM(absolute weights)`.
Just like the corresponding row-wise functions of fig.6a and fig.6b.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A		2		1.08					0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00
B		1		0.52						0.52					0.00	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.00
C		4		2.28		0.66			-0.55	0.67		-0.40			0.66	0.00	0.00	0.55	0.67	0.00	0.40	0.00	0.00
D		0		0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E		1		0.29		0.29									0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F		2		1.04			0.65			-0.39					0.00	0.65	0.00	0.00	0.39	0.00	0.00	0.00	0.00
G		0		0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H		2		0.76		-0.43				0.33					0.43	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
K		4		1.96			0.64		0.26	-0.63		0.43			0.00	0.64	0.00	0.26	0.63	0.00	0.43	0.00	0.00
						3	2	0	3	4	1	2	1	0	1.38	1.29	0.00	1.38	2.21	0.33	0.83	0.51	0.00

Figure 8 - The nodes' **IN** are in the green box below the weights and the nodes' **ID** are in the orange box below the absolute weights. these boxes will be transposed to the corresponding nodes column-wise.

The resulting table is shown in fig.8. Now we must *transpose*, i.e. switch the rows and columns of a copy of the table, then copy the transposed values as columns for **IN** and **ID**. The result is shown in fig.9.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A	3	2	1.38	1.08					0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00
B	2	1	1.29	0.52						0.52					0.00	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.00
C	0	4	0.00	2.28		0.66			-0.55	0.67		-0.40			0.66	0.00	0.00	0.55	0.67	0.00	0.40	0.00	0.00
D	3	0	1.38	0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E	4	1	2.21	0.29		0.29									0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F	1	2	0.33	1.04			0.65			-0.39					0.00	0.65	0.00	0.00	0.39	0.00	0.00	0.00	0.00
G	2	0	0.83	0.00											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H	1	2	0.51	0.76		-0.43				0.33					0.43	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
K	0	4	0.00	1.96			0.64		0.26	-0.63		0.43			0.00	0.64	0.00	0.26	0.63	0.00	0.43	0.00	0.00
						3	2	0	3	4	1	2	1	0	1.38	1.29	0.00	1.38	2.21	0.33	0.83	0.51	0.00

Figure 9 - How to transpose the **IN** and **ID** cells.

In the unfortunate case of a lack of transpose function of the spreadsheet, one should copy all the cells of fig.9 one by one, following the arrows order. Now we can compute **CEN,** the last node variable, which is simply the SUM() of **ID** and **OD**. The completed matrix is shown in fig.10.

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
A	3	2	1.38	1.08	2.46				0.57				-0.51		0.00	0.00	0.00	0.57	0.00	0.00	0.00	0.51	0.00
B	2	1	1.29	0.52	1.81					0.52					0.00	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.00
C	0	4	0.00	2.28	2.28	0.66			-0.55	0.67		-0.40			0.66	0.00	0.00	0.55	0.67	0.00	0.40	0.00	0.00
D	3	0	1.38	0.00	1.38										0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E	4	1	2.21	0.29	2.50	0.29									0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F	1	2	0.33	1.04	1.37		0.65			-0.39					0.00	0.65	0.00	0.00	0.39	0.00	0.00	0.00	0.00
G	2	0	0.83	0.00	0.83										0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H	1	2	0.51	0.76	1.27	-0.43				0.33					0.43	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
K	0	4	0.00	1.96	1.96		0.64		0.26	-0.63		0.43			0.00	0.64	0.00	0.26	0.63	0.00	0.43	0.00	0.00
						3	2	0	3	4	1	2	1	0	1.38	1.29	0.00	1.38	2.21	0.33	0.83	0.51	0.00

Figure 10 - Almost finished. All the numbers have been evaluated.

At last we can dispose of the extra rows and columns used for the calculations (fig.11 top). For the sake of finishing, let's order the nodes according to the following rules: 1. Descending **OUT**, 2. Ascending **IN** (fig.11 middle). Note that after the ordering the column and row order of the nodes names does not match!

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K
A	3	2	1.38	1.08	2.46				0.57				-0.51	
B	2	1	1.29	0.52	1.81					0.52				
C	0	4	0.00	2.28	2.28	0.66			-0.55	0.67		-0.40		
D	3	0	1.38	0.00	1.38									
E	4	1	2.21	0.29	2.50	0.29								
F	1	2	0.33	1.04	1.37		0.65			-0.39				
G	2	0	0.83	0.00	0.83									
H	1	2	0.51	0.76	1.27	-0.43					0.33			
K	0	4	0.00	1.96	1.96		0.64		0.26	-0.63		0.43		

	IN	OUT	ID	OD	CEN	A	B	C	D	E	F	G	H	K
C	0	4	0.00	2.28	2.28	0.66			-0.55	0.67		-0.40		
K	0	4	0.00	1.96	1.96		0.64		0.26	-0.63		0.43		
F	1	2	0.33	1.04	1.37		0.65			-0.39				
H	1	2	0.51	0.76	1.27	-0.43					0.33			
A	3	2	1.38	1.08	2.46				0.57				-0.51	
B	2	1	1.29	0.52	1.81					0.52				
E	4	1	2.21	0.29	2.50	0.29								
G	2	0	0.83	0.00	0.83									
D	3	0	1.38	0.00	1.38									

	IN	OUT	ID	OD	CEN	C	K	F	H	A	B	E	G	D
C	0	4	0.00	2.28	2.28					0.66		0.67	-0.40	-0.55
K	0	4	0.00	1.96	1.96					0.64		-0.63	0.43	0.26
F	1	2	0.33	1.04	1.37					0.65		-0.39		
H	1	2	0.51	0.76	1.27				0.33	-0.43				
A	3	2	1.38	1.08	2.46				-0.51					0.57
B	2	1	1.29	0.52	1.81							0.52		
E	4	1	2.21	0.29	2.50					0.29				
G	2	0	0.83	0.00	0.83									
D	3	0	1.38	0.00	1.38									

Figure 11 - ordering and finishing the matrix.

The columns should be re-arranged to match the order of the rows. This is why we marked the diagonal elements in the first place: just arrange the columns to rebuild the diagonal (fig.11 bottom). This new row arrangement naturally highlights the drivers (top rows) *C* and *K*, the receivers (bottom rows) *G* and *D*, and the remaining ordinary nodes *F, H, A, B* and *E* in the middle rows.

		IN	OUT	ID	OD	CEN	C	K	F	H	A	B	E	G	D
DRIVER	C	0	4	0.00	2.28	2.28					0.66		0.67	-0.40	-0.55
	K	0	4	0.00	1.96	1.96						0.64	-0.63	0.43	0.26
ORDINARY	F	1	2	0.33	1.04	1.37						0.65	-0.39		
	H	1	2	0.51	0.76	1.27			0.33		-0.43				
	A	3	2	1.38	1.08	2.46			-0.51						0.57
	B	2	1	1.29	0.52	1.81							0.52		
	E	4	1	2.21	0.29	2.50					0.29				
RECEIVER	G	2	0	0.83	0.00	0.83									
	D	3	0	1.38	0.00	1.38									

Figure 12 - The polished matrix.

The resulting matrix (fig.12) shows a clear block structure. The left and bottom blocks are necessarily empty, due to the ordering criteria adopted: since drivers cannot receive arrows the left block are empty, and since receivers cannot bear exiting arrows, the bottom blocks are empty. The top-right block represent the direct driver-receiver connections, while the central block represents the inner relationships, not directly connected with either drivers or receivers. Other types of ordering are possible, e.g. by centrality, but the resulting matrix lacks a clearly readable block structure.

Evaluating Model's Characters

The density, complexity and hierarchical index mentioned [Ö2004] can be easily computed by elementary spreadsheet techniques exploiting the matrix table obtained beforehand as described in the previous section.
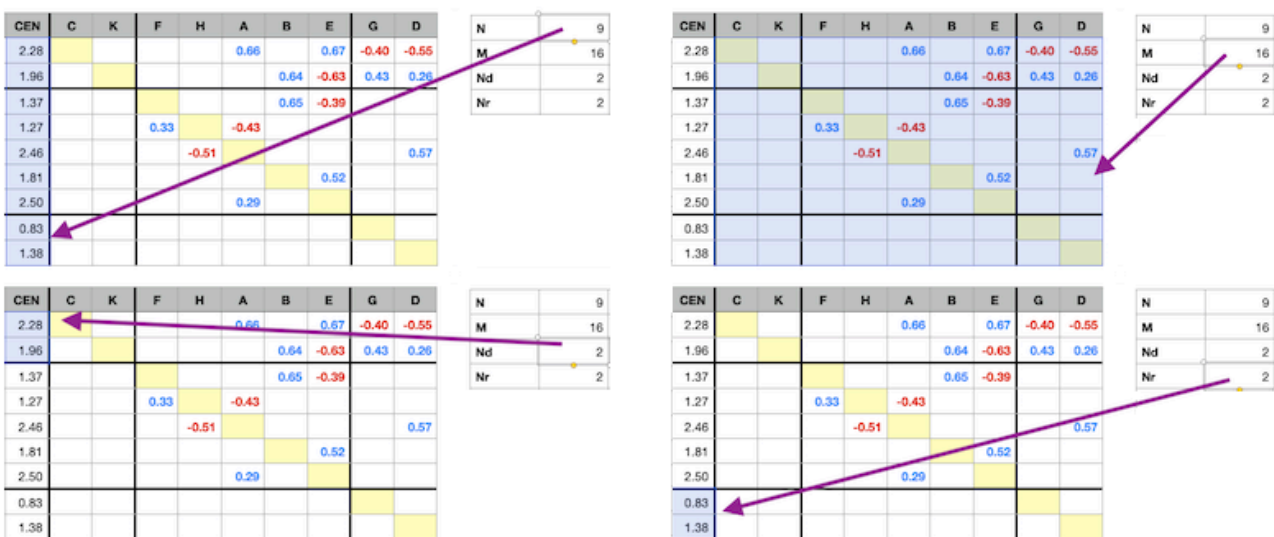

Figure 13 - Numbers evaluation via the COUNT() function.

Said N the number of nodes and M the number of connections, let's call N_d and N_r the number of drivers and receivers, respectively. These numbers are easily evaluated as the `COUNT()` of different cells of the weights matrix: N is `COUNT(CEN)`, M is `COUNT(weights)`, N_d is `COUNT(CEN, drivers only)` N_r is `COUNT(CEN, receivers only)`, see fig.13.The use of `COUNT()` on the centrality variable assures all non-zero values as input.

The evaluation of density and complexity is straightforward, it is just a matter of cell algebra.

$d = M/[N(N-1)]$ and $c = N_r/N_d$, while to calculate h we need a couple of intermediate steps:

Define a multiplicative factor $f = \dfrac{12}{(N+1)\,N\,(N+1)}$

Evaluate a temporary variable as the SUM(OD) $S = \displaystyle\sum_i od_i$

So that $h = f\displaystyle\sum_i \left(\dfrac{od_i - S}{N}\right)^2$. The latter summation can also be assigned to another temporary variable T.

		IN	OUT	ID	OD	CEN	C	K	F	H	A	B	E	G	D
DRIVER	C	0	4	0.00	2.28	2.28					0.66		0.67	-0.40	-0.55
	K	0	4	0.00	1.96	1.96						0.64	-0.63	0.43	0.26
ORDINARY	F	1	2	0.33	1.04	1.37						0.65	-0.39		
	H	1	2	0.51	0.76	1.27			0.33		-0.43				
	A	3	2	1.38	1.08	2.46			-0.51						0.57
	B	2	1	1.29	0.52	1.81							0.52		
	E	4	1	2.21	0.29	2.50					0.29				
RECEIVER	G	2	0	0.83	0.00	0.83									
	D	3	0	1.38	0.00	1.38									

N	9		
M	16	d	0.22
Nd	2		
Nr	2	c	1.00
f	0.0167	h	0.06
S	7.93		
T	3.55		

Figure 14 - Model density, complexity and hierarchical index.

Fig.14 shows the results of calculations. All the above can be eventually condensed in a simplified table, omitting the actual weights and the null values.

VARIABLE		IN	OUT	ID	OD	CEN
DRIVER	C		4		2.28	2.28
	K		4		1.96	1.96
ORDINARY	F	1	2	0.33	1.04	1.37
	H	1	2	0.51	0.76	1.27
	A	3	2	1.38	1.08	2.46
	B	2	1	1.29	0.52	1.81
	E	4	1	2.21	0.29	2.50
RECEIVER	G	2		0.83		0.83
	D	3		1.38		1.38

For the interpretation of the results see [MMM] and [Ö2004].

References

- [MMM] https://www.mentalmodeler.com/articles/Mental Modeler Manual for Workshop.pdf.
- [Ö2004] Uygar Özesmi and Stacy L. Özesmi, Ecological models based on people's knowledge: a multi-step fuzzy cognitive mapping approach, Ecological Modelling 176 (2004) 43-64, Elzevier B.V. doi:10.1016/j.ecolmodel.2003.10.027.

See also the references cited in https://www.mentalmodeler.com/#resources.

Credits

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