A general extrudate bulk density model for both twin-screw and single-screw extruder extrusion cooking processes

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Abstract

Effects of extrusion parameters and raw materials on extrudate expansion are respectively investigated in a twin-screw extruder and a single-screw extruder extrusion cooking experiments for fish feed, wheat, and oat & wheat mixture processing. A new phenomenological model is proposed to correlated extrudate bulk density, extrusion parameters and raw material changes based on the experimental results. The average absolute deviation (AAD) of the correlation is 2.2% for fish feed extrusion in the twin-screw extrusion process. For the single-screw extrusion process, the correlation AAD is respectively 3.03%, 5.14% for wheat and oat & wheat mixture extrusion; and the correlation AAD is 6.6% for raw material change effects. The correlation results demonstrate that the proposed equation can be used to calculate extrudate bulk density for both the twin-screw extruder and the single-screw extruder extrusion cooking processes.

Keywords: extrusion; expansion; model; fish feed; wheat

1. Introduction

Extrusion cooking is a thermo-mechanical food processing operation with an extruder. Inside the extruder, several processes may occur, including fluid flow, heat transfer, mixing, shearing, particle size reduction, and melting. Food extrusion is generally considered a high-temperature, short-time (HTST) process, where food materials are exposed to high temperatures for a very short time. This gives a distinct advantage over conventional pressure cooking, in which the exposure could be several minutes at temperatures near 100-140°C. The extrusion may have different objectives for different product productions. For example, the pasta manufacturing is to partially gelatinize starch, compact the dough, and give it the desired shape. In the case of breakfast cereals, flat bread and snacks production, an extruder is used to develop the desired expanded and porous structure. In this work, we focus on the expanded products in our experimental investigation and process modelling.

Fish feed or aquatic feed is also a type of expanded products processed by extrusion method. Extrusion technology plays an important role in fish feed manufacture for aquaculture industry. Aquaculture product production has a crucial role in many countries and areas. With increased demand for sustainable growth in aquaculture, alternative raw material sources have been recently tested to replace current ingredients as raw material in fish feed production due to the possible lack of fish meal supply in the future. To search for alternative fish feed recipes, a large amount of trails are needed to produce suitable products through extrusion processing. In the experimental investigation, a quantitative analysis and modelling for the extrusion process will aid engineers to efficiently produce suitable products for different types of fish feed.

In decades, many researchers continued to study the factors that affect the extrudate expansion in expanded product production, aiming to create the basis for new or improved extruded products. Moraru and Kokini (2003) reviewed a large amount of experimental and modelling work in this area. In these work, influence factors on extrudate expansion have been focused on cereal flours ingredients and physical properties, extrusion parameters, and screw configurations within an extruder. It should point out that screw configurations are critical for extrudate expansion, but they are not often changed on an existing extrusion line. Combining with introducing new products, recently, the effects of operational extrusion parameters on product quality have been intensively investigated (de Mesa, et al., 2009; Stojceska et al., 2009; Chakraborty et al., 2009; Wlodarczyk-Stasiak and Jamroz, 2009; Altan et al., 2009, 2008a, b; Wolf, 2010; Wójtowica and Mosciki, 2009; Chen, et al., 2010).

Modelling of extrusion process has been focused on understanding interactions between process parameters and product attributes (Moraru and Kokini, 2003). Klein and Marshall (1966) suggested a basic framework of mathematical models for extrusion cooking. Bruin et al. (1978) reviewed the extrusion process modelling work with that focusing on the relationship between flow pattern, flow rates, screw design and extrusion operation parameters. Mueser et al. (1987) proposed a system analytical model for extrusion cooking of starch. Alvarez-Martinez et al. (1988) suggested a general model for expansion of extruded products by taking into account dough moisture, melt temperature and die and screw shear strains. Eerikäinen and Linko (1989) reviewed the methods in extrusion cooking modelling, control and optimization. Kokini (1993) discussed a mechanism modelling strategy in quantitative characterization of extrusion process. Kokini suggested a detailed outline for establishment of the science base to make extrusion processing and extrudate quality predictable. Shankar and Bandyopadhyay (2004) developed a genetic algorithm to correlate the extrudate expansion with extrusion parameters. Ganjyal et al. (2003) explained the relationship between extrudate properties and extrusion parameters through neural network method. Numerical simulation and analysis have also been developed by researchers for the food extrusion process (Chiruvella et al. 1996; Li, 1999; Gonzalez et al., 2001; Weert et al., 2001; Dhanasekharan and Kokini 2003; Ficarella et al., 2006a, b; Alves et al. 2009, Tayeb et al., 1992). The numerical simulation often deals with the interactions between flow behaviour and extruder configuration, not for extrudate properties. In the modelling work, Response Surface Methodology (RSM) is also very often used (Altan et al. 2008a, b; Chakraborty et al. 2009, Chen, et al., 2010). The RSM results are very useful and practical. But RSM results are machine specific and have been limited to the scope of the specific investigations.

Another important methodology is to understand the food extrusion process in molecular level, e.g. starch degradation. The investigation results will not only explain extrudate bulk properties but also micro characteristics, such as molecular changes after processing. van den Einde et al. (2003) reviewed different research work in this area and also report their investigations (van den Einde et al., 2004). Brüemmer et al (2002) studied the effects of extrusion cooking on molecular weight changes of corn starch. It has been found that the molecular size of extruded starch, expressed as the weight average of the molecular weight (Mw), decreased exponentially when specific mechanic energy (SME) increased. The understanding of the starch changes in extrusion will finally result in the development of new food and biopolymer products.

Different from above methods, Cheng and Friis (2010) recently proposed a new phenomenological model from dimensional analysis and similarity principle (Buckingham, 1914; Stahl, 1962) to illustrate the interactions between extrusion parameters and extrudate expansion. The proposed bulk density model can well correlate extrudate expansion and extrusion parameters for different food and feed productions in a twin-screw extrusion process.

In this work, the bulk density model proposed by Cheng and Friis (2010) will be used to investigate the effects of extrusion parameters and raw material changes on extrudate expansion in a pilot scale twin-screw extruder for fish feed processing and a laboratory scale single-screw extruder for wheat, oat & wheat mixture extrusion. The experimental investigation, experimental data correlation and extrusion process modelling will be presented in the following sections.

2. Materials and Methods

2.1 Materials

2.1.1 *Twin-screw extruder extrusion experiment*

Raw materials for the experiments are fish meal, whole wheat flour and bean/pea flour mixture. The raw material composition changes are given in Table 1. The aim of the trials was to search for alternative raw materials to replace wheat and fish meal as a new generation sustainable aquatic feed. The fish meal was purchased from Skagen FF, wheat was obtained from Danish Agro. Beans were from DLF, and peas were from Danært. The wheat, beans and peas were obtained as whole grain and were ground. Beans and peas were fractionated by air classification to obtain a protein rich fraction. No further treatment was made for the ground wheat, bean and pea flours. The raw materials were mixed according to the recipes of Table 1 before addition into the volume feeder of the extrusion process. Experiments were designed to search for optimal expansion in extrusion processing fish feed according to small composite design method (Montgomery, 2001).

2.1.2 *Single-screw extruder extrusion experiment*

In the experiment, wheat flour and oat were purchased from supermarket (products of Lantmannen Mills A/S). The oat was ground in-house and without further treatment. Two recipes were prepared for the experiment: One is 100% wheat flour. Another is 70% (wt) oat flour and 30% (wt) wheat flour mixture. Experiments were designed to search for optimal expansion in extrusion processing wheat flour, oat and wheat flour mixture according to the small composite design method (Montgomery, 2001).

2.2 Extruder and extrusion experiment

2.2.1 *Twin-screw extruder extrusion experiment*

A Werner & Pflederer Continua 37 co-rotating twin-screw extruder (Coperion GmbH, Stuttgart, Germany) was used for the trials. The screw configuration was not changed during the experiments. The extruder has five zones with independent heating and cooling capabilities. The first zone (starting from the feed point) temperature was held at room temperature (ca. 25°C). The zone 5 temperature (closest zone to die) was set as the key control parameter. The zone 2-4 temperatures were set several degrees higher than the temperature of zone 5. A volumetric twin-screw feeder was used to feed raw materials into the first zone. Water was added by a dosing pump into the first zone. Two circular dies having a diameter of 3 mm were installed with the extruder. The recorded data during extrusion trials were zone 1-5 temperatures, heating oil temperatures in different zones, feed flowrate, added water flowrate, screw speed, die pressure, moment, die temperature and blade speed. The moment is converted to a torque value with an equation provided by the extruder producer.

The independent extrusion variables were varied in the following ranges: barrel temperature: 80-110°C, feed water content: 15-30 % (wt), screw speed: 280-380 rpm. The extrudates were cut by a two-blade adjustable knife assembly rotating at 550 rpm and conveyed to a horizontal belt dryer (Lytzen, HBD812). The extrudates were dried at 110°C in the dyer for 15 min to final moisture content of 5-8 % (wt). The extruder settings were allowed to achieve and maintain a steady state condition for approximately 10 min prior to sample collection.

After drying, the bulk density was measured by a one litre cup. Extrudates were filled in the one litre cup. The weight was recorded and the bulk density was calculated according to the weight of one litre extrudates. The bulk density results were the average of three readings for three cup measurements. The moisture content of the extrudates was determined in three replicates by drying approximately 1.5 g of each sample in an infrared dryer at 160 °C until constant weight. The experimental data of the trials are given in Table 2. A total of 20 runs were carried out for the fish feed extrusion.

2.2.2 *Single-screw extruder extrusion experiment*

A laboratory scale single-screw extruder, Haake Rheomex 19/25 (Thermo Fisher Scientific, Inc.) was used in the experiment. The screw diameter is 19.05mm, L/D=25. The extruder has three zones with independent heating and cooling capabilities. The first zone (starting from the feed point) temperature was held at around 50°C to heat raw material. The die temperature was set as a key control parameter. The zone 2-3 temperatures were set at the same value as the die temperature. One circular die having a diameter of 3 mm was installed in the extruder. The recorded data during extrusion experiment were zone 1-3 temperatures, product flowrate, screw speed, die pressure, torque and die temperature. Here, the product flowrate is not the same as the flowrate inside extruder as water vapour is escaped during discharging the product from die. The collected extrusion product was cut by a cutter to about 6-10mm in length. The 6-10mm long extrudates were dried in a cooking oven (Rational, CCC101/02, Germany) at 120°C for 25 min. The extruder settings were allowed to achieve and maintain a steady state condition for about 5 min prior to sample collection. After drying, the extrudate bulk density was measured by weight- volume- method. Extrudates were filled in a cup. The weight of the filled cup was recorded and the bulk density was calculated according to the weight of extrudates. The volume of the cup was calibrated with distillate water at room temperature. The bulk density results were the average of three readings for three cup measurements.

The experiments were designed from small composite design method (Montgomery, 2001). Water was mixed with flours in advance for all samples according to the small composite design result. The total water contents of all the pre-prepared samples were calibrated by following AACC method 44-15A (AACC, 2000). Three independent parameters were used in the experimental design, i.e. die temperature, water content and screw speed. After initial screening for the extrusion parameters, the independent parameters were varied in the following ranges: die temperature: 150 - 260°C, adding water content: 15 - 22 %, screw speed: 130 - 250 rpm.

2.3 Experimental results

Table 2 presents the experimental results in the twin-screw extruder, and Table 3-4 for the results in the single-screw extruder. In Table 2-4, T_d represents the die temperature, °C P_d represents the die pressure, bar, F_T represents the total flow rate of all inlet materials (flours and water), kg/hr, τ represents the torque, Nm, X_w represents the water content, g/g, X_{wh} represents the wheat content in fish feed raw material, g/g, N_s represents the screw speed, rpm, ρ_B^{exp} represents the bulk density of extrudates, g/L (g/litre).

3. Theory

3.1 Model development strategy

In an extrusion process, many factors have impact on extrusion product properties, such as cereal flours ingredients and physical properties, extrusion parameters, and screw configurations within an extruder, etc. As discussed in section 1, efforts have been made to develop a model to represent the correlations between extrusion parameters and extrudate bulk density. Among of these efforts, response surface methodology (RSM) is a common used way to correlate extrusion parameters and extrudate bulk density. In the RSM investigations, some extrusion parameters are widely selected in experimental design, correlation model development and impact factor analysis. To take advantage of the RSM research work, we will use dimensional analysis method to evaluate the common selected parameters in the RSM investigations, such as extrusion temperature, SME, water content, etc. Then, we will develop a model to involve only the most important parameters. Such model is a phenomenological equation to approximate the system behaviours and may not give a complete image for the system.

3.2 Model development

3.2.1 *Science meets art in dimensional analysis practice*

Dimensional analysis is a technique that has been applied in engineering modelling for a long time (Stahl, 1962; Langhaar, 1951). Historically, dimensional analysis can be done using the Rayleigh method or the Buckingham pi method. In this work, we follow the Buckingham pi method. The Buckingham pi method gives a solution for a general problem in finding the minimum number of variables necessary to define the relationship between *n* variables. These variables are expressible in terms of *k* independent fundamental physical quantities, then the original expression is equivalent to an equation involving a set of $p = n - k$ dimensionless variables constructed from the original variables. However, the Buckingham pi theorem does not provide a rigours way to select these physically meaningful variables. Here the science meets the art: the selection of the physically meaningful variables is highly subjective and arbitrariness, beyond any rigorous rules. As the RSM has been often used in extrusion process analysis, the common selected variables in RSM are very helpful in the variable selections.

3.2.2 *The work of Cheng and Friis (2010)*

Based on the dimensional analysis principle, Cheng and Friis (2010) have developed a phenomenological equation to correlate extrudate density and extrusion parameters. However, the effects of raw material changes on extrudate expansion are not included in the model of Cheng and Friis (2010). Thus, we will try to develop a correlation to represent the effects of raw material changes on extrudate bulk density.

In the work of Cheng and Friis (2010), extrudate bulk density is expressed through three dimensionless process parameter groups as follow:

$$\left(\frac{F_W}{F_T}\right)^{\alpha}\left(\frac{T_d}{T_0}\right)^{\beta}\left(\frac{P_d \cdot F_T}{\rho_B \cdot \tau \cdot N_s}\right)^{\gamma} = K \qquad (1)$$

where F_T is the total flowrate of all inlet materials (flours and water) in an extruder, kg/hr, F_w is the water flowrate added into the extruder, kg/hr, T_d is the die temperature, °C, T_0 is the zone 1 (feed zone) temperature, which is used to represent raw material initial temperature and given a fixed value of 25 °C, N_s is the screw speed, rpm, τ is the torque, Nm, P_d is the die pressure, bar, $\bar{\rho}_B$ is the bulk density of extrudates, g/liter, α, β, γ, K are dimensionless coefficients determined from experimental data.

In equation (1), the group F_w/F_t represents the water content of the processing material (not including the natural moisture content of grain flour). The group T_d/T_0 is the temperature changes of the processing material from its initial condition to the vicinity of die before discharging from extruder. The group $\frac{P_d \cdot F_T}{\bar{\rho}_B \cdot \tau \cdot N_S}$ represents the extrusion pump efficiency of the extrusion process, which has a conversion constant, 104/6 by using above units. Within the group, the term $\tau \cdot Ns/F_T$ is the widely used specific mechanical energy (SME).

For convenience in regression with experimental data, equation (1) is modified as:

$$\bar{\rho}_B = K(X_w)^\alpha \left(\frac{T_d}{T_0}\right)^\beta \left(\frac{P_d \cdot F_T}{\tau \cdot N_s}\right)^\gamma \tag{2}$$

In equation (2), $X_w = F_w/F_T$ represents the water content of processing material.

As can be seen from equation (1), many important extrusion process properties are not involved in the equation, such as raw material composition and physical properties, material flow properties, extruder dimensions, etc. The equation (1) does not intend to represent the complete extrusion cooking behaviours, but only for extrudate bulk density.

3.2.3 *Model formation*

The first step in the dimensional analysis is to select suitable variables to setup the study system. In the variable selection, we take advantage of the RSM results in extrusion cooking process applications. To include raw material changes into equation (1), we use raw material composition as a variable to represent the changes. The selection of composition is arbitrary. More physical meaningful raw material properties may be employed for such purpose. As shown in Table 1, three different components, i.e. wheat, bean/pea and fish meal flour, are involved in the recipes. To simplify the final model, we chose only one component to represent the composition changes. The component is wheat flour flowrate in raw material. The wheat flour flowrate and other selected variables are given in Table 5.

The second and third steps are to determine the rank of dimension matrix and to form the model. Using dimension analysis principle (Langhaar, 1951), the dimension matrix of the selected process parameters is given in Table 6. From Table 6, the rank of the dimension matrix is calculated, which is 5. As can be seen from Table 5, the number of the selected independent variables is 9. Thus, 4 dimensionless groups should be formed to build a dimensionless expression. In this work, the dimensionless expression is given as follow.

$$K = \left(\frac{F_w}{F_T} \frac{F_{wh}}{F_T} \right)^{\alpha} \left(\frac{T_d}{T_0} \right)^{\beta} \left(\frac{P_d \cdot F_T}{\rho_B \cdot \tau \cdot N_s} \right)^{\gamma} \qquad (3)$$

where F_{wh} is the flowrate of wheat flour in the raw material flow, kg/hr, all other symbols have the same meanings as equation (1). For simplification, the dimensionless groups F_w/F_T and F_{wh}/F_T are arranged together with one experimental determined parameter in equation (3). It should be point out that the group F_{wh}/F_T can be the composition of any adjusting component (ingredient) in the recipes shown in Table 1. For practical application in regression with experimental bulk density data, equation (3) is rearranged as

$$\rho_B = K\left(\frac{F_v}{F_T}\frac{F_{vh}}{F_T}\right)^{\delta}\left(\frac{T_d}{T_0}\right)^{\theta}\left(\frac{P_d \cdot F_T}{\tau \cdot N_s}\right)^{\omega} \quad (4)$$

where K is not a dimensionless constant, K, δ, θ and ω are experimentally determined parameters.

It should be pointed out that the formation of equation (4) is to reduce the difficulties in regression with experimental data and may convert the equation to an empirical model. Thus, equation (4) is an approximation of equation (3). Any other type of algebraic expression or simply a graphical relation among these three dimensionless groups that accurately fits the experimental data would be an equally valid manner of the model. As Box (Box and Draper, 1987) pointed out ―Essentially, all models are wrong, but some are useful‖, we need to validate the model with experimental data.

4. Results

4.1 Regression results for the twin-screw extrusion experiment

Equation (4) is used to correlate experimental bulk density data. Taking the experimental data of Table 2, the model coefficients K, δ, θ and ω are determined with following objective function.

$$\text{obj} = \min \sum_{i=1}^{n}\left[\bar{\rho}_{B,i}^{\text{exp}} - \bar{\rho}_{B,i}^{\text{cal}}\right] \quad (5)$$

The obtained coefficients are given in Table 7. In the regression, the conversion constant (104/6) is not used in the calculation as ω is an experimental determined parameter. To search possible global minimum, random initial values with different signs are used in the regression. 500 set of initial values were produced. MATLAB software (The Math Works) was used in the model coefficient determination. The average absolute deviation (AAD) of the model correlation for the experimental bulk density data is 2.2%, where AAD is calculated as.

$$\mathrm{AAD} = \frac{1}{n}\sum_{n}\left[\frac{\left|\rho_B^{\exp} - \rho_B^{\mathrm{cal}}\right|}{\rho_B^{\exp}}\right]\% \qquad (6)$$

In equation (6), n is the number of experimental runs. The model correlation results for the extrudate bulk density are represented in Figure 1. As shown in Figure 1, the model can well correlate the experimental data. Among the 20 experimental points, however, 5 points have ca. 5% deviation in the correlation. The proposed equation cannot completely represent the exact behaviours of the extrudate bulk density but only an approximation.

4.2 Regression results for single-screw extrusion experiment

Equation (2) is used to correlate experimental bulk density data and process parameters for wheat, and oat & wheat mixture extrusion, respectively. Taking the experimental data of Table 3-4, the model coefficients K, α, β and γ are determined with equation (5) as objective function. The obtained coefficients are given in Table 7. The AAD of the model correlation for the experimental bulk density data are respectively 3.03% and 5.14% for wheat flour, and oat & wheat mixture extrusion. The model correlation results for the extrudate bulk density are represented in Figure 2-3.

To treat the experimental data in Table 3-4 as one set of data, i.e. one recipe is wheat flour, the other is 70% (wt) oat and 30%(wt) wheat mixture, equation (4) is employed to correlate all these data in Table 3-4. The AAD of the regression is 6.6%. The correlation results are given in Figure 4.

As shown in Figure 2-4, equation (2) and (4) can well correlate extrudate bulk density and extrusion parameters in the single-screw extrusion system. However, the equations cannot capture the behaviours for some points. In figure 4, 5 points have the deviation more than 10%. In figure 3, 3 points have the deviation more than 10%. The deviations probably come from the equations as they are only a phenomenological approximation for the relationship between extrudate bulk density and extrusion parameter. More mechanism research is needed for such relationship. From the correlation results, we can say the bulk density model (equation (2) and (4)) can be employed for both twin-screw and single-screw extrusion process. The consistent general equation will bring a new way to calculate the extrudate bulk density for complete different extruders.

5. Discussion

The model development takes advantage of RSM in important variable selections to carry our dimensional analysis. The equation (4) is simply a new expression for all these well-know extrusion parameters. To include more important extrusion parameters, such as flour rheological property, starch gelatinization degree, screw configuration, etc., a more general equation for any raw flour variations may be obtained. However, our aim is to develop an equation only for determining the extrudate bulk density based on the phenomenological way.

In general, response surface method (RSM) and principle component analysis (PCA) are very suitable and efficient in operation parameter determination and optimization. However, the RSM and PCA method often cannot result in a consistent equation to describe the extrusion processing for different extruders and raw material recipes. It is difficult to use the RSM results to design a new process operation. Therefore, a general consistent analytical equation is needed for the different extrusion cooking processes. The consistent equation is very necessary for extrusion experimental design, extrusion process design, scale-up, extruder design, extrusion process control and optimization.

It has been shown in section 4 that the proposed equation can be used to correlate different flours extrusion in the different extruders without modification for its formula. Such characteristic may demonstrate that the proposed equation can determine the impacts of those important parameters on extrudate bulk density for different flour extrusions in the twin-screw and single-crew extruder extrusion processes.

In study of influence factors on extrudate expansion, one often needs to look at the effect of an individual factor on extrudate expansion, such as initial water content vs. bulk density, etc. Using equation (2), we can study the impact of extrusion parameters on extrudate bulk density. To take logarithm for equation (2), we have

$$\ln \bar{\rho}_B = \ln K + \alpha \ln X_w + \beta \ln\left(\frac{T_d}{T_0}\right) + \gamma \ln\left(\frac{P_d \cdot F_T}{\tau \cdot N_s}\right) \qquad (7)$$

The extrudate bulk density is expressed in term of the sum of different extrusion operational parameters. From a set of experimental data, we can observe the contribution of an operational parameter to extrudate bulk density.

5.1 Extrusion process operation simulation--Twin-screw extruder extrusion

One basic application of equation (4) is to quantitatively analyse the obtained experimental data in order to observe the effects of extrusion parameters on extrudate expansion. Using equation (4) to simulate the extrusion experiment, the interactions between different operational parameters and the contributions of these parameters to extrudate bulk density can be taken into account numerically at different recipe conditions. With the model parameters (Table 7), the simulation is carried out for the fish feed extrusion. In the simulation, the experimental boundary conditions are taken from Table 2. The boundary ranges of the dimensionless groups in the simulation are set as T_d/T_0=3.4-4.6, $P_d \cdot \tau \cdot Ns/F_T$ =P_d/SME =0.05-0.2, X_{H2O}=16-24%, X_{wheat}=2-21%. The simulation results are given in figure 5-6. In figure 5-6, three extrusion operation surfaces are plotted at water content, 16%, 20% and 24%. From 5-6, we can see that the change of water content has no strong effects on bulk density in the fish feed extrusion as the three operation surfaces are almost merged each other. Figure 5-6 also shows that if the bulk density around 450-550 g/l is set as the observation scope, the corresponding operation parameter region or window is moving with recipe changes. The operational region of recipe 3 moves to lower P_d/SME region comparing to the control sample. As shown in Table 2, the screw speed is higher in processing of recipe 3 than that of the control sample. The lower P_d/SME means that a higher SME is needed for recipe 3 processing in the extrusion.

5.2 Effects of extrusion parameters on extrudate bulk density—Single-screw extruder extrusion

Taking the correlation coefficients from Table 7, the effects of three operational or adjustable parameters on bulk density are represented in Figure 7-8 using equation (7). From Figure 7-8, it can be seen that the contributions of operational extrusion parameters to bulk density have the same tendency. The contribution of $-\gamma \ln(N_s)$ or $\alpha \ln X_w$ to bulk density is nearly a constant in the two experiments. The most sensitive factor is temperature in the two experiments. The die temperature can be set as a key control parameter.

5.3 Operation surface of extrusion process—Single-screw extruder extrusion

Taking the correlation coefficients from Table 7, we plot operation surface using equation (2) for wheat, and oat & wheat extrusion. In the plot, the boundary conditions for T_d/T_0 , $P_d \cdot \tau \cdot Ns/F_T = P_d$/SME and water content are taken from the experimental data in Table 3-4. The results are presented in Figure 9-10. In Figure 9-10, three operation surfaces are plotted at 3 different water contents from 20-30%. Among the three operation surfaces, the minimum water content operation surface, X_{H2O}=20%, is located at the lowest position for both wheat and oat & wheat mixture extrusion. To compare the operation surface in figure 9-10, it shows that a local minimum bulk density value exists with a suitable Pd/SME value. Higher die temperature will give good expansion (lower bulk density) for both cases. If we set a bulk density target value for the extrusion process, we can identify a suitable operation region from the analysis of Figure 9-10.

6. Conclusions

In this work, we developed a new phenomenological equation to study the effects of raw material changes on extrudate bulk density from dimensional analysis methodology. The model is employed to correlate the experimental extrudate bulk density data from fish feed extrusion in a twin-screw extruder and wheat and oat & wheat mixture extrusion in a single-screw extruder. The correlation results show that the proposed equation can well determine the extrudate bulk density for different flour extrusions in both twin-screw and single-crew extruder extrusion processes. However, the bulk density model cannot precisely capture the extrudate bulk behaviours for some experimental points in the two extruder extrusion processes. Using the bulk density model (equation (2) and (4)), we have analysed the fish feed, wheat, and oat & wheat mixture extrusion characteristics in the two extruders. The general extrudate bulk density model provides a consistent analytical equation to describe the extrudate expansion behaviours in different extruder extrusion cooking processes. The model development method may establish a way to search for more generalized understanding for the extrudate expansion and other properties.

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Figure 1

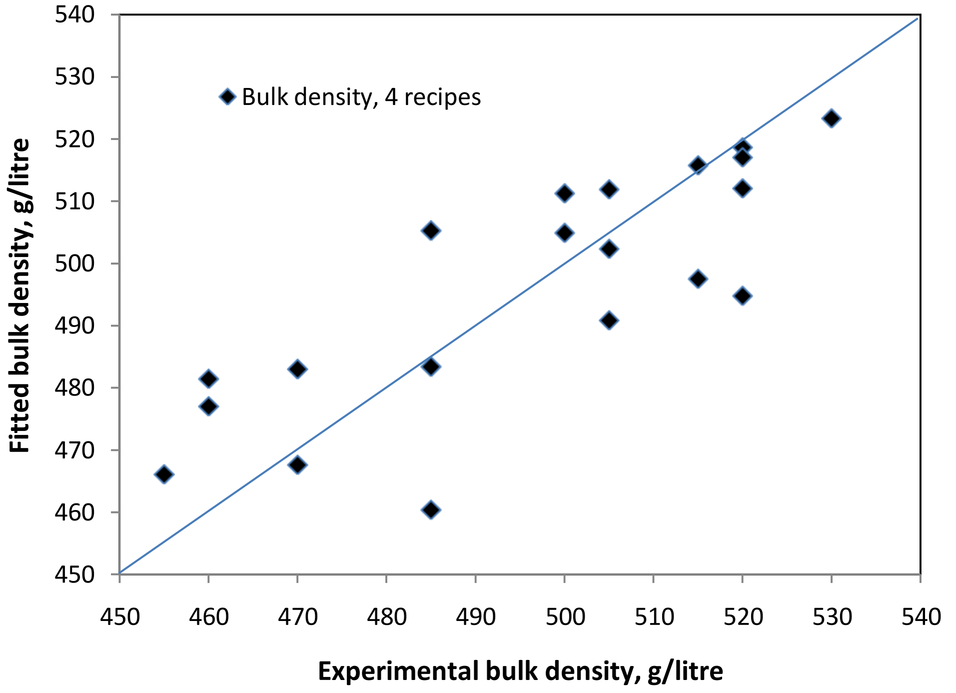

Figure 1 Correlation results of equation (4) for fish feed extrusion in the twin-screw extruder

Figure 2

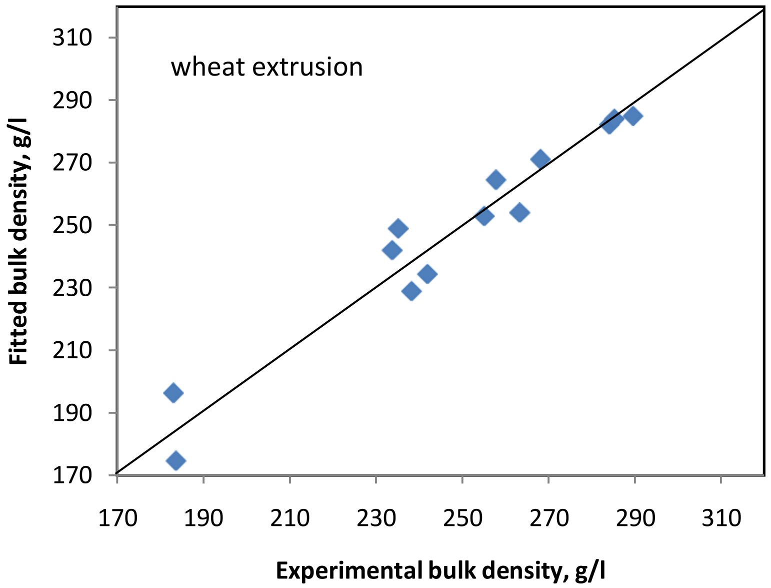

Figure 2 Correlation results of equation (2) for wheat flour extrusion in the single-screw extruder

Figure 3

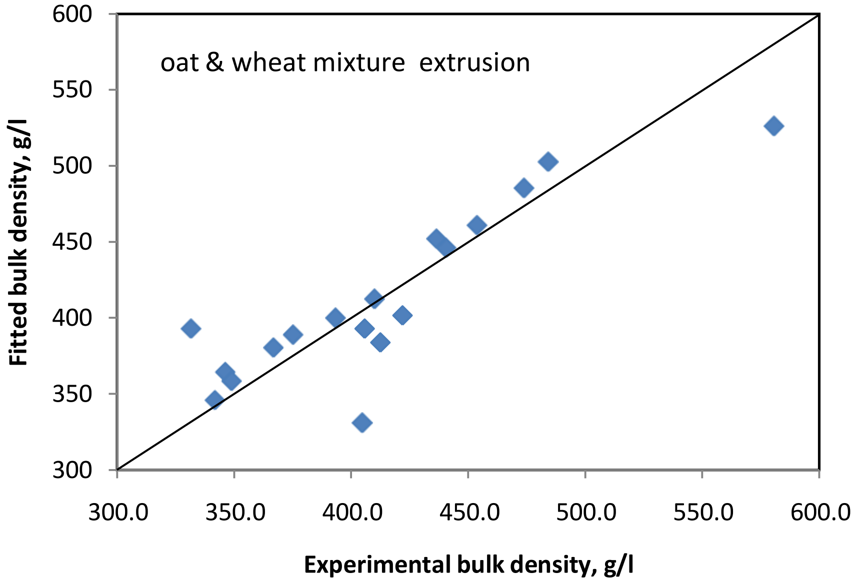

Figure 3 Correlation results of equation (2) for oat & wheat mixture extrusion in the single-screw extruder

Figure 4

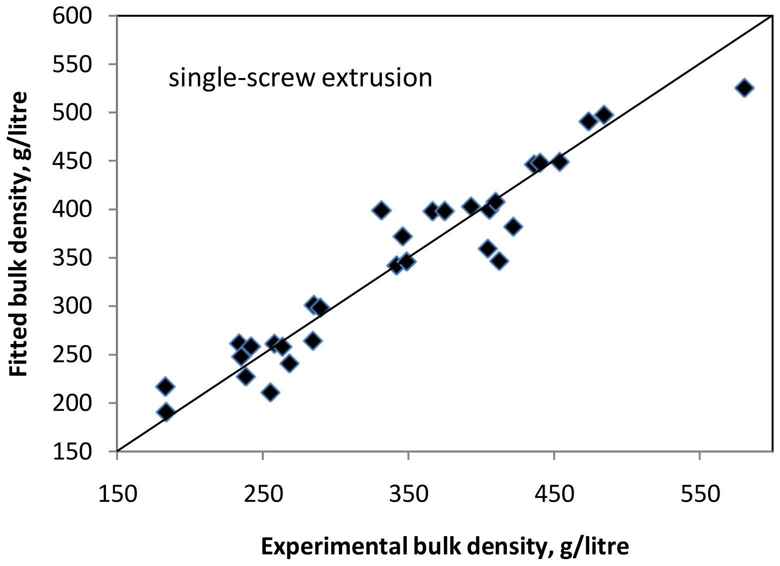

Figure 4 Correlation results of equation (4) for all data in Table 3-4

Figure 5

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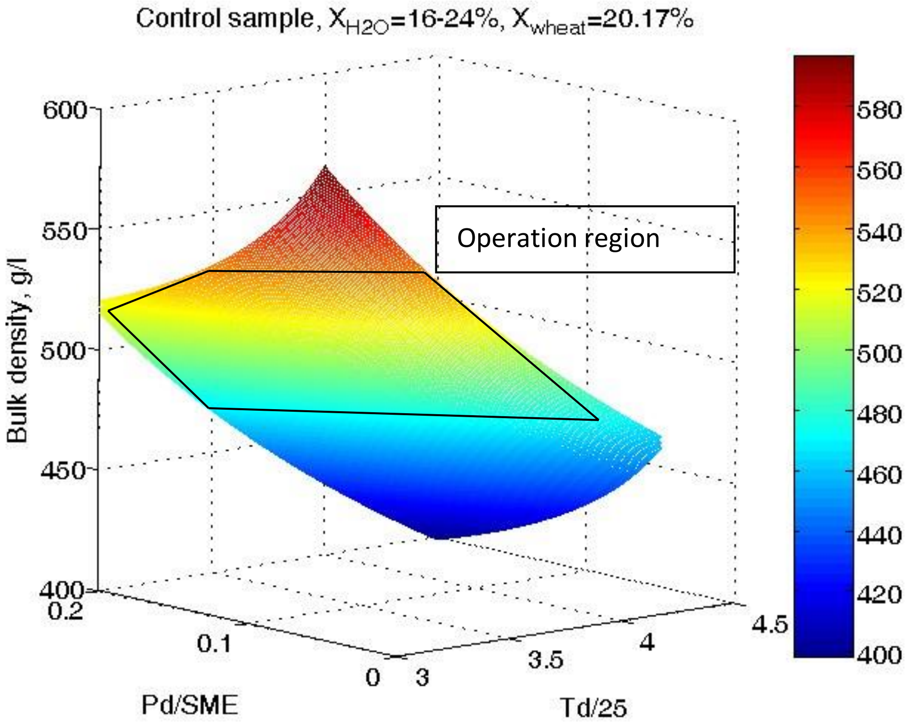

Figure 5 Fish feed extrusion processing simulation results (twin-screw extruder) at X_{H2O}=16-24%, X_{wheat}=20.17%, control recipe

Figure 6

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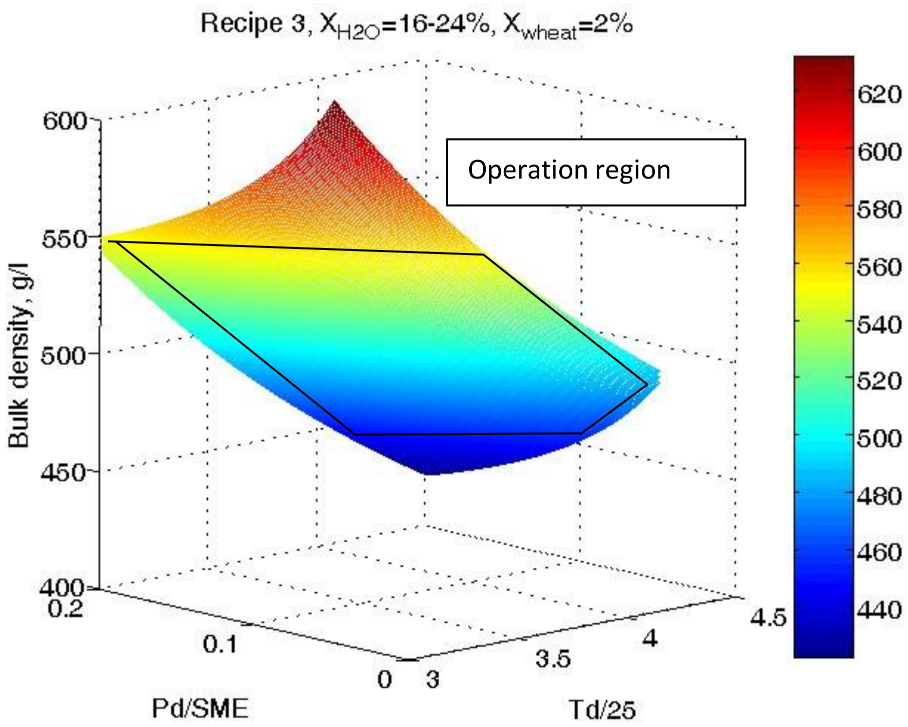

Figure 6 Fish feed extrusion processing simulation results (twin-screw extruder) at X_{H2O}=16-24%, X_{wheat}=2%, recipe 3

Figure 7

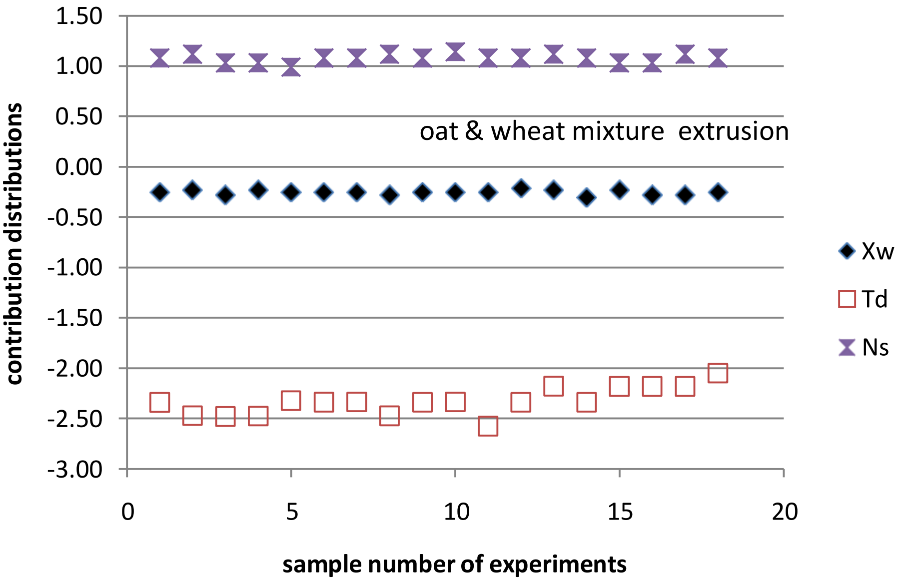

Figure 7 Effects of operation parameters on bulk density, oat & wheat mixture extrusion (single-screw extruder). ◆ Xw is the $\alpha \ln X_w$, ☐ Td is the $\beta \ln\left(\frac{Td}{T_0}\right)$, × Ns is the $-\gamma\ \ln(N_s)$ in equation(7)

Figure 8

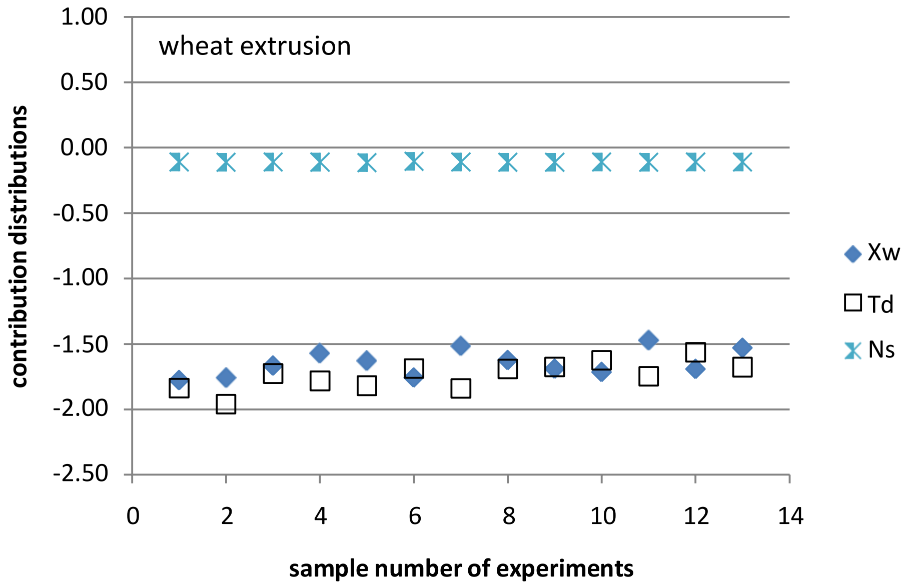

Figure 8 Effects of operation parameters on bulk density, wheat extrusion (single-screw extruder). ◆ Xw is the $\alpha \ln X_w$, ☐ Td is the $\beta \ln\left(\frac{Td}{T_0}\right)$, × Ns is the $-\gamma \ln(N_s)$ in equation (7)

Figure 9

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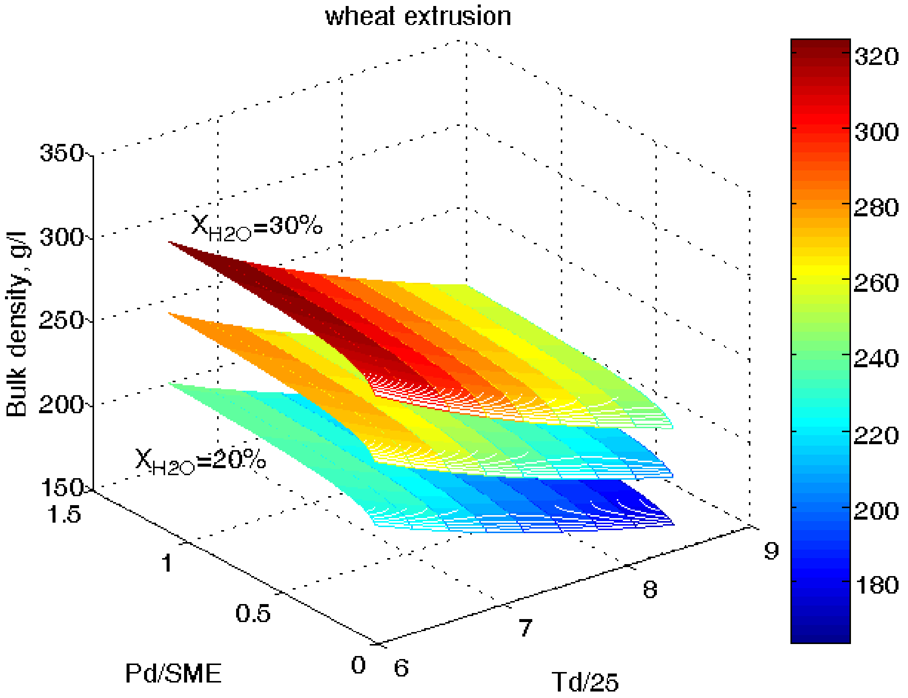

Figure 9 Extrusion operation surface at different water contents, wheat extrusion (single-screw extruder)

Figure 10

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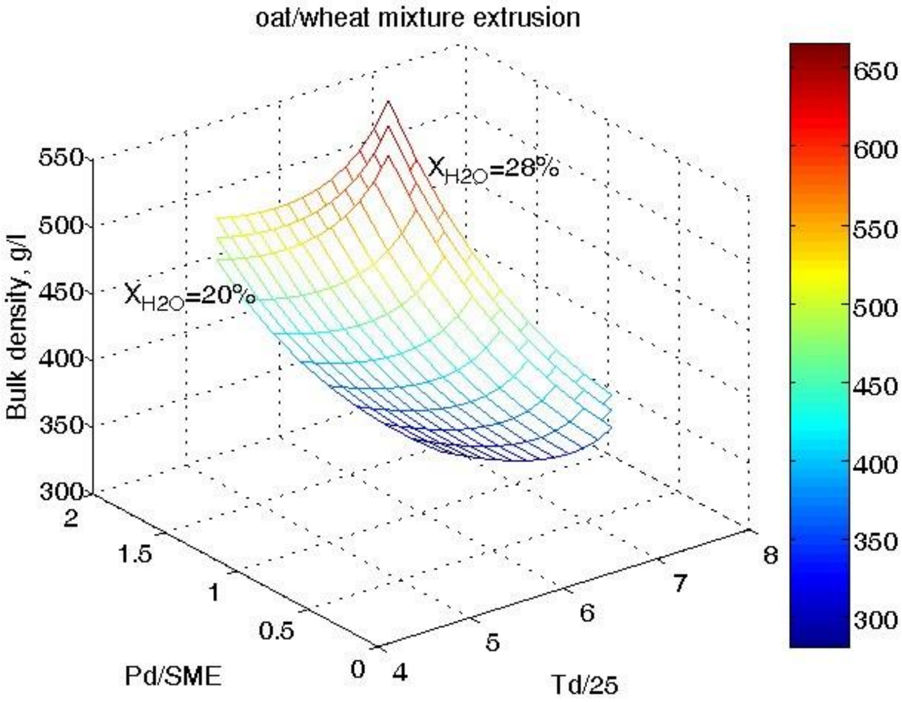

Figure 10 Extrusion operation surface at different water contents, oat & wheat mixture extrusion (single-screw extruder)

Figure captions

Figure 1 Correlation results of equation (4) for fish feed extrusion in the twin-screw extruder

Figure 2 Correlation results of equation (2) for wheat flour extrusion in the single-screw extruder

Figure 3 Correlation results of equation (2) for oat & wheat mixture extrusion in the single-screw extruder

Figure 4 Correlation results of equation (4) for all data in Table 3-4

Figure 5 Fish feed extrusion processing simulation results (twin-screw extruder) at X_{H2O}=16-24%, X_{wheat}=20.17%, control recipe

Figure 6 Fish feed extrusion processing simulation results (twin-screw extruder) at X_{H2O}=16-24%, X_{wheat}=2%, recipe 3

Figure 7 Effects of operation parameters on bulk density, oat & wheat mixture extrusion (single-screw extruder). ◆ Xw is the $\alpha \ln X_w$, □ Td is the $\beta \ln\left(\frac{T_d}{T_0}\right)$, × Ns is the $-\gamma \ln(N_s)$ in equation(7)

Figure 8 Effects of operation parameters on bulk density, wheat extrusion (single-screw extruder). ◆ Xw is the $\alpha \ln X_w$, □ Td is the $\beta \ln\left(\frac{T_d}{T_0}\right)$, × Ns is the $-\gamma \ln(N_s)$ in equation (7)

Figure 9 Extrusion operation surface at different water contents, wheat extrusion (single-screw extruder)

Figure 10 Extrusion operation surface at different water contents, oat & wheat mixture extrusion (single-screw extruder)

Table 1

Table 1 Composition of different recipes

	Control	Recipe 1	Recipe 2	Recipe 3
Fish meal, wt%	58.90	51.02	43.14	35.22
Wheat, wt%	20.17	14.08	7.99	2.00
Bean+pea, wt %	0.0	13.66	27.32	41.00

Table 2

Table 2 Fish feed extrusion experimental results

Sample	T_d °C	P_d bar	F_T kg/hr	τ Nm	X_w g/g	X_{wh} g/g	N_s rpm	ρ^{exp} g/L
Control	95	32	27.50	32.85	0.20	0.2017	269	460
	99	33	27.70	34.31	0.21	0.2017	277	455
	97	19	26.82	27.74	0.25	0.2017	320	520
	98	32	33.00	37.96	0.20	0.2017	278	470
	98	39	34.15	37.96	0.18	0.2017	291	485
	96	21	29.47	32.85	0.23	0.2017	277	505
Recipe 1	102	17	26.46	24.82	0.2396	0.1408	299	460
	98	22	28.24	27.01	0.2245	0.1408	299	470
	88	26	27.53	28.47	0.2372	0.1408	301	520
	92	44	29.69	32.85	0.1748	0.1408	328	485
Recipe 2	86	32	27.49	29.2	0.2033	0.0799	325	530
	88	34	26.38	28.47	0.2039	0.0799	340	515
	90	34	26.41	29.2	0.1746	0.0799	337	500
	91	35	25.99	31.39	0.1920	0.0799	310	485
	91	31	25.11	27.74	0.1987	0.0799	345	505
	93	31	26.80	25.55	0.1791	0.0799	370	505
Recipe 3	92	30	31.88	28.47	0.1688	0.0200	345	520
	93	28	31.82	27.01	0.1750	0.0200	355	520
	99	24	29.38	28.47	0.1961	0.0200	321	500
	99	25	31.93	30.66	0.2045	0.0200	294	515

Table 3

Table 3 Oat & wheat mixture extrusion experimental results (70% (wt) oat, 30% (wt) wheat)

Sample	T_d °C	P_d bar	F_T kg/hr	τ, Nm	X_w g/g	N_s rpm	ρ^{exp} g/L
1	166.2	16.21	5.92	4.6	0.295	223	341.8
2	150.2	17.08	5.40	7	0.260	185	331.5
3	120.2	19.11	1.66	7.58	0.260	185	580.8
4	132.7	22.12	4.79	6.46	0.295	223	436.5
5	133.0	18.08	4.34	6.07	0.225	147	473.9
6	167.1	23.65	4.85	5.33	0.225	147	346.2
7	150.1	19.97	2.81	5.11	0.200	185	440.5
8	149.7	16.25	5.25	3.38	0.260	185	393.3
9	133.0	18.82	6.53	4.77	0.225	223	484.2
10	149.6	18.01	4.94	5.64	0.260	250	409.9
11	166.3	27.99	6.07	8.01	0.225	223	404.6
12	149.8	16.24	3.54	5.11	0.260	185	375.0
13	150.0	13.50	6.32	6.07	0.260	185	405.7
14	148.2	10.85	1.58	3.95	0.260	120	366.8
15	133.0	12.67	4.54	4.49	0.295	147	453.8
16	166.7	17.03	2.61	7.85	0.295	147	348.8
17	179.8	5.90	2.53	6.89	0.260	185	412.4
18	150.0	15.72	5.68	3.53	0.320	185	422.0

Table 4

Table 4 Wheat flour extrusion experimental results

Sample	T_d °C	P_d bar	F_T kg/hr	τ Nm	X_w g/g	N_s rpm	ρ_{exp} g/L
1	160.5	27.77	5.85	0.79	0.255	190	268
2	170.3	46.66	8.01	9.48	0.260	225	263
3	192.4	10.72	4.25	6.05	0.286	210	235
4	184.9	7.35	2.76	3.59	0.309	225	284
5	170.5	6.61	2.28	3.8	0.296	190	290
6	200.6	12.09	5.73	4.51	0.273	250	238
7	180.4	9.26	4.50	3.72	0.265	190	234
8	235.8	19.30	9.09	13.182	0.246	225	184
9	173.2	33.35	10.41	14.87	0.274	225	258
10	172.6	39.18	7.44	11.90	0.247	155	242
11	149.8	51.30	7.98	11.85	0.260	190	285
12	204.7	25.45	7.89	9.97	0.243	190	183
13	205.3	14.39	5.97	5.10	0.299	190	255

Table 5

Table 5 Selected extrusion process parameters

Parameters	Symbols	Units	Dimensions
Die temperature	T_d	°C	T
Zone 1 temperature	T_0	°C	T
Total inlet flowrate	F_T	kg/hr	M/t
Added water flowrate	F_W	kg/hr	M/t
Screw speed	N_s	1/min	$1/t$
Torque	τ	Nm	$F \cdot L$
Die pressure	P_d	bar	F/L^2
Bulk density	$\bar{\rho}_B$	g/liter	M/L^3
Wheat flour flowrate	W_{wh}	kg/hr	M/t

Table 6

Table 6 Selected variables and unit

Units	T_0	T_d	F_T	F_w	N_s	P_d	τ	$\bar{\rho}_B$	F_{wh}
T	1	1	0	0	0	0	0	0	0
M	0	0	1	1	0	0	0	1	1
t	0	0	-1	-1	-1	0	0	0	-1
F	0	0	0	0	0	1	1	0	0
L	0	0	0	0	0	-2	1	-3	0

Table 7

Table 7 Experimental determined parameters for equation (2) and (4)

Parameters	*K*	δ	θ	ω
Fish feed extrusion	814.62	-0.02446	-0.6307	-0.09922
	K	α	β	γ
Wheat extrusion	7633.6	1.2553	-0.8741	0.0207
Oat & wheat extrusion	3097.6	0.1885	-1.3045	-0.2070