Valuing Multidimensional Environmental Changes with Contingent Ranking*

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Abstract

Assessment of multidimensional environmental change requires specialized valuation methods, of which contingent ranking is one candidate. Luce's Choice Axiom permits an easy linkage between ranking and choice through the Cascading Choice Theorem which states that a ranking of alternatives is equivalent to a sequential choice process. The Reversibility Paradox shows this to be an untenable assumption. In light of this is the contingent ranking method modified to an experimental procedure which explicitly elicits an incomplete ranking of the alternatives through a sequential choice process. The theoretical and statistical properties of such a valuation method are discussed.

1 Introduction

When developing and analyzing policies that have environmental impacts there is a need for knowledge about the trade-off between the different environmental characteristics. This inherent multidimensionality of environmental change has important implications for the choice of valuation method.

The contingent valuation method is widely used for valuing both simple and complex packages of environmental goods. However, only with careful specification of the contingent valuation experiment is it possible to estimate a multidimensional trade-off surface for the different environmental characteristics. Reliable estimation of the surface with conventional contingent valuation methods requires large and complex survey samples (Hoehn 1991).

Contingent ranking (Rae 1983), the little known sibling of contingent valuation, is in many aspects better suited for valuing multidimensional environmental trade-offs than contingent valuation. The contingent ranking method differs from contingent valuation in that the respondent in the valuation experiment is asked to rank a large number of alternatives with combinations of environmental goods and prices as compared to the two alternatives presented in the referendum format of contingent valuation. The complete ranking of the alternatives is then subjected to statistical analysis, and implicit prices imputed (Rae 1983, Lareau and Rae 1989).

The contingent ranking method has met with mixed responses (Smith and Desvousges 1986, Lareau and Rae 1989). Implementations of contingent ranking have typically involved the ranking of large numbers of alternatives which often appear similar to the respondent. The cognitive task of arriving at a complete ranking is experienced by the respondents as very difficult and demanding task. The estimated statistical models of the ranking data are often poor which results in imprecise environmental values.

In this paper I take a closer look at the contingent ranking method and develop a simpler, and theoretically more defensible, version of contingent ranking. The idea behind the contingent ranking method is an appealing one, but closer scrutiny of the method will show that the linkage between ranking and choice behavior is an uneasy one which places severe and unrealistic restrictions on the models. Recognizing this, I propose to modify the experimental setup of the contingent ranking experiment such that the response elicitation process imposes a particular heuristic consistent with choice behavior. The resulting response data is then consistent with an explicit choice model which can be analyzed within the standard framework of probabilistic choice behavior.

The paper is organized with a brief review of contingent valuation and ranking in the next section. The relationship between choice and ranking behavior is then discussed at length, and the notion of censored rankings introduced. The following section proposes a revised contingent ranking method and discusses implementation and estimation of values for this method.

2 Stated Preference Experiments

There are a number of methods available for determining stated preferences through experiments. Especially within psychology exists a substantial research tradition on this topic.[1] There is also increasing interest in, and use of, general experimental methods in economics (Davis and Holt 1993).

The motivation for this research is in the estimation of preferences for complex environmental goods and the valuation of multidimensional changes in environmental amenities and services. Thus the intentions are not to provide a comprehensive review of stated preference experiments, but rather to indicate how they are, or can be, used in non-market valuation studies.

[1] One important line of research starts with the work of Thurstone (1927), continuing with Coombs (1964), Krantz, Luce, Suppes, and Tversky (1971) and Suppes, Krantz, Luce, and Tversky (1989).

2.1 Nonmarket Valuation Methods

Economists have devised a number of methods for assigning a value, or price, to those goods and services not routinely traded in functioning markets.[2] The nonmarket valuation methods can be divided into two braod categories:

1. revealed preference methods, and
2. stated preference methods.

Revealed preference methods are widely used for valuation of many types of nonmarket goods. Important classes of such methods are hedonic pricing models and models based on preference interdependencies such as weak complementarity.

2.2 Contingent Valuation

The dominant nonmarket valuation method based on stated preference experiments is the *contingent valuation* method (Mitchell and Carson 1989). The (modern) roots of the method goes back to Davis (1963), Randall, Ives, and Eastman (1974) and Brookshire, Ives, and Shulze (1976).

The basic idea behind contingent valuation is to ask the participants in the valuation experiment more or less directly about the willingness-to-pay for a hypothetical change in the bundle of environmental goods and services. The elicited value is contingent upon the scenario specified in the experiment.

Although contingent valuation is an expanding research program (Hoehn and Randall 1987), certain features are emerging as standard.[3] A status quo with respect to institutions and the provision level of one or more non-market goods is described to the contingent valuation experiment participant; an alternative level is proposed; and then, within well-specified conditions under which the alternative level will be provided and

[2] Pearce and Markandya (1989) and Bergstrom (1990) give brief introductions to different nonmarket valuation techniques along with references to the pertinent literature. See Freeman (1993) or the volume by Braden and Kolstad (1991) for a state of the art review.

[3] The current debate in the US of the status of contingent valuation and the pending regulations from NOAA regarding guidelines for the use of contingent valuation methods in damage assessment are inteded to bring forth *one* standard for conducting valuation studies. These guidelines may not always prescribe appropriate approachs for all valuation experiments.

individual payments collected, does the investigator elicit the participant's contingent valuation according to a preselected method. Methods that elicit the participant's continuous valuation response — whether by an open ended "how much would you pay" question or by some type of bidding game — have been popular in the past. In the referendum approach the investigator posits a single offer and records the participant's "yes" or "no" response. These responses are typically analyzed with dichotomous choice models (logit or probit) from which a welfare measure can be derived (Bishop and Heberlein 1979, Hanemann 1984, Cameron 1988, Mitchell and Carson 1989).

2.3 Contingent Ranking

Contingent ranking is an alternative method to contingent valuation proposed in the early eighties (Rae 1983). The method is implemented much in the same way as contingent valuation. However, the method differs from contingent valuation in that the respondent in the experiment is asked to rank order a large number of alternatives with combinations of environmental goods and prices as compared to the two alternatives given in the referendum format of contingent valuation.

The data on the complete ranking of all the alternatives is then analyzed using a random utility function framework. The estimation is often done with the econometric technique of Beggs, Cardell, and Hausman (1981), which is essentially a multinomial logit model of the rank order of the random utility level associated with each alternative. Implicit attribute prices or welfare change measures are then calculated from the parameter estimates of this logit model.

The contingent ranking method has met with mixed responses (Cummings, Cox, and Freeman 1986, Smith and Desvousges 1986, Lareau and Rae 1989). The implementations of contingent ranking experiments have typically involved the ranking of large numbers of alternatives which often appear very similar to the respondent. The cognitive task of arriving at a complete ranking is often experienced as a difficult and demanding task. The final statistical model of the stated rankings is often poor which results in questionable price estimates.

2.4 Conjoint analysis

Conjoint analysis is a method widely used in marketing studies, although with strong disciplinary roots in psychology and statistics (Luce and Tukey 1964, Kruskal 1965, Green and Srinivasan 1978). From the point of individual choice theory, as used in economics, does the theoretical foundation of conjoint analysis seem rather shaky (Madansky 1980, McFadden 1986, Bates 1988). There is, however, a trend in conjoint analysis from reliance on pure statistical methods towards more behaviorally based models such as the multinominal logit model (McFadden 1986, Louviere 1988).

One feature of conjoint analysis is that each individual participating in the conjoint analysis experiment is faced with a large number of ranking tasks. Each ranking task involves a small number of alternatives, typically only two. Based on the collected data, some type of utility index model is estimated for *one* individual. This differs from contingent valuation and ranking where a large number of individuals are asked about their stated preferences for one set of alternatives, and a *representative* random utility model is estimated for the relevant population.

The strength of conjoint analysis is in the explicit use of statistical experimental design techniques to explore a number of different attributes in a choice or ranking setting. However, this design feature need not be limited to the conjoint analysis, but could be linked with more general random utility models of choice behavior (Hensher 1982, Bates 1988, Louviere 1988).

2.5 Valuation of Complex Environmental Goods

The multidimensional environmental impacts of many policies require estimation of multidimensional trade-off surfaces. Although contingent valuation can be modified through careful survey design to include controlled changes in several environmental characteristics, this require complex experimental setups involving large number of splits in the survey sampling framework (Hoehn 1991). Reliable estimation of multidimensional trade-off surfaces will require a large number of respondents in the sample.

The actual experimental design for valuation of multidimensional environmental change can draw upon the available methods used in conjoint analysis. Building upon the strengths of contingent valuation and conjoint analysis makes contingent ranking a viable alternative for valuation of multidimensional environmental change.

3 Models of Choice and Ranking

Based upon the Bradely-Terry-Luce model (Bradley and Terry 1952, Luce 1959) of ranking data and individual choice it is possible to formulate *random utility* models of ranking and choice data (Block and Marschak 1960, Marschak 1960). Using Luce's choice axiom (Luce 1959) and the cascading choice theorem of Luce and Suppes (1965), ranking data can be transformed into choice data (Chapman and Staelin 1982). That is, the alternative given rank one is the choice when all alternatives are available, the alternative given rank two is the choice when all alternatives except the alternative given rank one is available, etc.

This decomposition of rankings into a sequence of choices is often needed for estimation of preferences. The estimation procedure of Chapman and Staelin (1982) requires this decomposition in order to permit estimation of preferences with a multinominal logit model. Beggs, Cardell, and Hausman (1981) instead imposes distributional assumption on a model of rankorders which leads to the same multinominal logit model. Although these are observationally equivalent models, they differ substantially in their conceptual underpinnings and in the interpretation of assumptions.

The purpose of this section is to explore the use of the Bradely-Terry-Luce model to ranking data, and especially to examine in some detail the validity of decomposition of the ranking data into choice data and the ensuing use of multinominal logit models.

3.1 Notation

Let A be the universal set of alternatives, and suppose that an individual faces a finite set of choices, $C \subseteq A$. Let $P_C(c)$ denote the probability that c is chosen from an available set C of alternatives. If $S \subseteq C$, then $P_C(S)$ denotes the probability that the selected alternative is in the subset S.

The function $P_C(\cdot)$ defines a standard probability measure on the subsets of C for a fixed choice set C (Chung 1974), i.e.

Axiom 1 (Probability Measure)

1. *For all* $S \subset C$*,* $0 \leq P_C(S) \leq 1$.

2. $P_C(C) = 1$.

3. *If* $R, S \subset C$ *and* $R \cap S = \emptyset$*, then* $P_C(R \cup S) = P_C(R) + P_C(S)$.

The following result concerning summation of probabilities for mutually exclusive events is well-known from the theory of probability

$$P_C(S) = \sum_{s \in S} P_C(s). \tag{1}$$

The probability measure defined on C constitutes a structure of choice probabilities which is closed for finite A. Such structures of choice probabilities can be analyzed either with a *strict utility model* or a *random utility model*, see the discussion in Luce and Suppes (1965) or Suppes, Krantz, Luce, and Tversky (1989).

Definition 1 *A closed structure of choice probabilities satisfies the* strict utility model *iff there exists a positive real-valued function* ψ *on* A *such that for all* $c \in C \subseteq A$,

$$P_C(c) = \frac{\psi(c)}{\sum_{s \in C} \psi(s)}. \tag{2}$$

Definition 2 *A closed structure of choice probabilities satisfies the* random utility model *iff there exists a collection* $\mathcal{U} = \{u_a : a \in A\}$ *of jointly distributed random variables such that for all* $c \in C \subseteq A$,

$$P_C(c) = \Pr\{u_c \geq u_s \quad \forall s \in C\}. \tag{3}$$

Thus the probability of observing that a particular alternative c is chosen equals the probability that this alternative has the greatest utility value. This gives the random utility model a linkage to order statistics (Critchlow, Fligner, and Verducci 1991).

3.2 Luce's Choice Axiom

Some of the most important theoretical work concerning choice and ranking takes the Choice Axiom of Luce (1959) as its point of departure.

Axiom 2 (Choice Axiom)
A closed structure of choice probabilities, with $P_C(S) \neq 0$ *for all* $S \subseteq C$, *satisfies the* choice axiom *iff for all* $T \subseteq S \subseteq C$ *the following holds*

$$P_C(T) = P_S(T)P_C(S). \tag{4}$$

The choice axiom asserts basically that the choice process leading to the selection of T (or an element of T) from the total set C of available alternatives can be decomposed into independent choices:

1. the choice of T from S, and
2. the choice of S from C.

There are a number of consequences of the Choice Axiom of which only a few will be repeated here.[4] The following theorem is often taken as an alternative statement of the choice axiom.

Theorem 1
The choice axiom implies that the following holds for any $s, c \in C$

$$\frac{P_{\{s,c\}}(s)}{P_{\{s,c\}}(c)} = \frac{P_C(s)}{P_C(c)}. \tag{5}$$

This result is known alternatively as the *constant ratio* rule or the *independence from irrelevant alternatives* property of choice. It is a difficult and restrictive feature of the choice axiom and one which is not always reasonable (Debreu 1960, McFadden 1976, McFadden 1981).

Theorem 2
The choice axiom implies that for A *and its subsets, there exists a positive real-valued function* v *on* A*, which is unique up to multiplication by a positive constant, such that for every* $C \subset A$

$$P_C(c) = \frac{v(c)}{\sum_{s \in C} v(s)}. \tag{6}$$

Hence, choice processes which satisfies the choice axiom can be rationalizable with some strict utility function.

As far as random utility model goes, it is commonly believed that the choice axiom (because of the independence from irrelevant alternatives) implies a multinominal logit type of random utility model. The following theorem shows that this is not exactly the case (Yellot 1977, Strauss 1979).

Theorem 3
Let the random variables $(u_a; a \in A)$ *be independently distributed with a common distribution function* F*. Then the choice probabilities will satisfy the choice axiom if and only if* $(u_a; a \in A$ *is type I extreme value (or double exponential), i.e.*

$$F(x) = e^{-e^{-\frac{x-\alpha}{\beta}}}. \tag{7}$$

[4]See Luce and Suppes (1965) and Luce (1977).

The additional assumption of independently distributed random variables must be added in order to link the choice axiom with the logit model. Of course, if the random utilities are assumed to be independently and identically distributed then the logit model is inevitable (McFadden 1973, Strauss 1979).

3.3 Rankings

Thus far the concern has been with choice, but this modelling framework is extendable to the question of ranking. The exposition here follows to a large extent the review in Colonius (1984).

Consider the finite set of alternatives $C = \{c_1, c_2, \ldots, c_n\}$ where all the alternatives are indexed from 1 to n, and let $\mathcal{R}_C$ be the set of all possible permutations of the elements in C. One such permutation is $\rho \in \mathcal{R}_C$ where

$$\rho = (c_{i_1}, c_{i_2}, \ldots, c_{i_n})$$

is the permutation of the elements in C which gives alternative c_{i_1} rank number one, c_{i_2} rank number two, etc.. Let $\rho(i)$ denote the rank of alternative c_i, and thus $\rho^{-1}(j)$ is the index of the alternative with rank j. Let $r(\rho)$ denote the probability of observing for one individual the rankorder ρ.

We are now in position to define a probability measure on rankorders, i.e. the probability of a particular alternative having a specified position in the rankordering. In the random utility approach an individual rank alternatives in the same order as the corresponding utility variables, i.e.

$$\begin{aligned}
\{c_{i_1}\} &= \big\{c \in C : u_c > u_s \quad \forall s \in C \setminus \{c\}\big\}, \\
\{c_{i_j}\} &= \Big\{c \in C \setminus C_j : u_c > u_s \quad \forall s \in C \setminus \big(C_j \cup \{c\}\big)\Big\} \quad j = 2, \ldots, n-1, \\
\{c_{i_n}\} &= C \setminus \{c_{i_1}, \ldots, c_{i_{n-1}}\} = C \setminus C_n
\end{aligned} \tag{8}$$

where

$$C_j = \{c_{i_1}, \ldots, c_{i_{j-1}}\} \quad \text{for } j = 2, \ldots, n.$$

The properties of such probability measures are given by the following result due to Block and Marschak (1960).

Definition 3 *For a ranking on any set* C *the ranking probabilities are given by*

$$\Pr(\rho(i) = j) = \sum_{\tau \in R_{i,j}} r(\tau) \tag{9}$$

where $R_{i,j} = \{\rho \in \mathcal{R}_C : \rho(c_i) = j\}$.

3.4 Decomposition

It is now possible to give a more precise characterization of relationship between choice and ranking in terms of decomposing the ranking to a sequence of choices. In particular it is taken to be a requirement that choices and rankings are consistent with each other (Block and Marschak 1960).

Condition 1 *The* consistency condition *(between choice and ranking) states that a structure of choice probabilities satisfies a random utility model iff there exists a probability measure on rankings such that*

$$P_C(c_i) = \sum_{\rho \in R_i^*} r(\rho) \tag{10}$$

where $R_i^* = R_{i,1} = \{\rho \in \mathcal{R}_C : \rho(c_i) = 1\}$.

Condition 2 Complete decomposition *implies that the probability of a ranking of alternatives can be written as the probability of a sequence of choice*

$$r(c_i, c_j, \dots, c_k, c_l) = P_{\{c_i, c_j, \dots, c_k, c_l\}}(c_i) P_{\{c_j, \dots, c_k, c_l\}}(c_j) \cdots P_{\{c_k, c_l\}}(c_k). \tag{11}$$

The following relationships between decomposition and the choice axiom exist

Theorem 4
For a random utility model

1. *complete decomposition implies the choice axiom,*
2. *the choice axiom implies decomposition for alternative sets with exately three alternatives.*

Theorem 5
Let the random variables $(u_a; a \in A)$ *be independently distributed with a common distribution function* F*. Then the choice probabilities will satisfy complete decomposition if and only if* F *is double exponential.*

This is a version of the cascading choice theorem of Luce and Suppes (1965). Thus the choice axiom in itself is not strong enough to provide the necessary structure to make choice and ranking interchangeable. Additional assumptions about the distribution of the random errors are needed, and again i.i.d. assumptions are typical, which immidiately leads to the multinominal logit model (Beggs, Cardell, and Hausman 1981, Chapman and Staelin 1982, Ben-Akiva, Morikawa, and Shiroishi 1991). However, if we accept a common distribution function on the stochastic part of the random utility model, then our resulting model will satisfy the choice axiom, ensure consistency between choice and ranking, and permit a tractable statistical specification of the model.

In terms of rankings are there both theoretical and empirical evidence supporting the view that the link between choice and ranking breaks down as the rankorder is traversed (Chapman and Staelin 1982, Ben-Akiva, Morikawa, and Shiroishi 1991). A complete ranking of many alternatives may not be the best implementation of ranking experiments, but rather one should use an incomplete ranking requesting the few best alternatives and, possibly, the worst alternatives.

3.5 Reversibility

Block and Marschak (1960) showed the surprising result that under the assumptions of Luce's Choice Axiom will a ranking of three alternatives in terms of the best would yield the same ranking in terms of the worst only in the special case of indifference between the three alternatives. There is by now a large literature on this topic, see Yellot (1980) or Colonius (1984) for a review. .

Let $P^*_C(c)$ denote the probability that alternative $c \in C$ is picked as the worst alternative among those available. Following Marley (1968) the quantitity $P^*_C(c)$ is termed the *aversion probability*. Furthermore, let $r^*(\rho)$ denote the probability that an individual ranks, in terms of the worst, the alternatives in C in the sequence ρ.

Theorem 6 (Reversibility Paradox)
Let n=3 and assume the Choice Axiom, then for each $\rho \in \mathcal{R}_C$, $r(\rho) = r^*(\rho)$ if and only if

$$P_{\{1,2,3\}}(i) = P^*_{\{1,2,3\}}(i) = \frac{1}{3} \quad i = 1, 2, 3,$$

and

$$P_{\{i,j\}}(i) = P^*_{\{i,j\}}(i) \;\; = \frac{1}{2} \quad i,j = 1,2,3; \quad i \neq j.$$

Thus it must be concluded that the Luce Choice Axiom is incompatible with the inutitively sensible requirement that ranking probabilities should be essentially the same whether the subjects rank from best to worst or vice versa.

Two alternative routes exists for dealing with the reversibility paradox. As noted by Yellot (1980), the paradox is rooted in the fact that the double exponential probability distribution function is asymmetric. Thus one approach is to explore the use of alternative distributions which is consistent with some behavioral model, preferably a random utility model, and which yields decomposable rankings. An additional benefit of this approach is the potential to avoid the problem of independence from irrelevant alternatives which is implied by the use of the multinominal logit model. On the other hand, it is not computationally feasible to estimate anything but the multinominal logit model for choice situations involving more than a few alternatives.

Another route is to design the elicitation experiment such as to force a uni-directional preference ordering throughout the elicitation process. Given the evidence about the heuristics in use when solving cognitive problems (Eysenck and Keane 1990) it may be difficult and costly to ensure that the assessment technique employed in an experimental setting is monotone from best to worst, or vice versa.

3.6 Incomplete rankings

A *partial* ranking of the k best alternatives among the n alternatives in a set $C \subseteq A$ is denoted $\rho_{k/n}$ where

$$\rho_{k/n} \in \left\{ (c_{i_1}, \ldots, c_{i_n}) : c_{i_m} \in \{c \in C : k < \rho(c)\},\; m = k+1, \ldots, n \right\}.$$

An *incomplete* ranking of the k best alternatives and the l worst alternatives among the n alternatives in a set $C \subseteq A$ is denoted $\rho_{(k,l)/n}$ where

$$\rho_{(k,l)/n} \in \left\{ (c_{i_1}, \ldots, c_{i_n}) : c_{i_m} \in \{c \in C : k < \rho(c) < l\},\; m = k+1, \ldots, l-1 \right\}.$$

Under the assumptions of independently distributed random utilities from a double exponential distribution function will applications of probability calculus yield that the probability of a particular incomplete ranking $\rho_{(k,l)/n}$ is

$$\Pr(\rho_{(k,l)/n}) = \left(\prod_{j=1}^{k} P_{\{c_{i_j},\dots,c_{i_n}\}}(c_{i_j})\right)\left(\prod_{j=k+1}^{l-1} \sum_{S\in\mathcal{S}(j,l)} P_S(c_{i_j})\right)\left(\prod_{j=l}^{n} P_{\{c_{i_j},\dots,c_{i_n}\}}(c_{i_j})\right) \quad (12)$$

where

$$\mathcal{S}(j,l) = \left\{(c_{i_j},\dots,c_{i_n}) : c_{i_m} \in \{c \in C : j \le \rho(c) < l\},\ m = j,\dots,l-1\right\}.$$

The only difference between this probability for an incomplete rankorder and that for a sequential choice model is in the middle term which takes into account the different permutations of the choice set consistent with the observed incomplete ranking.

4 Design of Incomplete Ranking Experiments

The discussion in the previous section leads to the conclusion that there are potentially serious problems when formulating theoretical and statistical models for contingent ranking data. This section pursues the idea of structuring the contingent ranking experiment in such a way that only parts of the ranking of all the available alternatives are elicited sequentially and then in an explicit choice-like process, thereby imposing a choice heuristic on the cognitive task of evaluating the available alternatives.

This section explains in more detail the setup of an incomplete contingent ranking experiment.[5]

4.1 Experimental Design

A characteristic feature of multidimensional environmental change is that the change affect several environmental characteristics at once. The task is to estimate the utility surface spanning these characteristics. There is a long tradition in statistics and marketing science in the design of experiments which efficiently identifies and estimates

[5]As contingent ranking as a valuation method shares many similarities with contingent valuation a number of the survey design techniques used in contingent valuation studies applies to contingent ranking as well (Mitchell and Carson 1989, Cummings, Cox, and Freeman 1986). These issues will not be discussed any further here.

the parameters defining multidimensional trade-off surfaces (Montgomery 1984, Louviere and Woodworth 1983, Fowkes and Wardman 1988). One of the strengths of conjoint analysis is the explicit reliance on statistical experimental design techniques to construct different bundles of characteristics to use in product comparisons.

A factor, as the term is used in the experimental design literature, is some kind of treatment to be applied in the experiment. Thus, change in one environmental characteristic is one factor, and a specified magnitude of this change is the level of that factor. One of the factors to be included in a contingent ranking experiment is the price, or cost, of the environmental change.

A factorial experiment is an experimental design which includes all possible combinations of the levels of all the factors. The number of combinations can be extremely large for factorial experiments and hence prohibitively costly to implement. Fractional factorial experiments is a widely used method for reducing the number of combinations to be considered in an experiment (McLean and Anderson 1984). The cost of using fractional factorial experiments is that some high-order effects, say third- or even some second-order effects in some designs, cannot be identified and estimated.

Even with fractional factorial designs the number of combinations may become excessively large, and intractable in a ranking experiment. In conjoint analysis this is often handled by splitting the task into a number of pairwise comparisons (Bissett and Schneider 1991). Another approach is to use a blocked fractional factorial design in the experiment. A block in a contingent ranking number is a number of different combinations of the treatment levels randomly assigned to that unit, i.e. questionnaire. Different blocks are randomly assigned to each participant.

4.2 Choice Elicitation

A participant in the ranking experiment faces a number of different environmental impact scenarios, including stipulated costs (or bid prices). In the incomplete contingent ranking experiment the participant is asked to consider all the presented alternatives and then state which one these alternatives is considered best. This alternative is then removed and the participant asked to state which of the remaining alternatives is best. This is repeated k times. The participant is then asked which of the remaining alternatives is the worst. This alternative is then removed and the task is performed a total l times. Thus, at the end of the choice elicitation process does the experimenter have available the $\rho_{(k,l)/n}$ incomplete ranking of the n presented alternatives.

This explicit use of the choice elicitation task in the experiment ensures that the data is collected in a manner consistent with a probabilistic choice model, and thus circumvent the need for relying on decomposition of the (complete) ranking in the analysis and estimation.

5 Estimation of Value

The starting point for analysis of preferences and valuation of environmental change is individual behavior, and I start with a review of an explicit model of individual choice behavior under quantity rationing. Such models captures the essence of the implications of changes in environmental characteristics for individual welfare.

5.1 Individual Choice Behavior Under Quantity Rationing

A useful conceptual framework for modeling individual choice behavior when environmental commodities are present is to assume that the individual regards those commodities not traded in markets as fixed in quantity. This abstraction leads to the notion of quantity rationing. Under *quantity rationing* an individual is able to freely choose a consumption level for some (nonrationed) goods, but is allotted certain amounts of other (rationed) goods. To sell any of the allotment is precluded and it cannot be supplemented by additional purchases. Pollak (1969) calls these rationed goods "preallocated". The rationing levels an individual faces may be specific for that individual, specific for a geographical region, or the same for all individuals in a nation. The important characteristic is that the individual regards the quantity of these commodities as outside of her direct control in the short run.

The consumption by a consumer is an l-list of the quantities of the various commodities she consumes. Her consumption bundle is a vector, $\boldsymbol{\omega}$, in the commodity space $\mathbf{E}^l$. Let the set of possible consumption bundles for the consumer, denoted $\boldsymbol{\Omega}$, be a nonempty, closed, convex subset of $\mathbf{E}^l_+$, .

The commodity vector $\boldsymbol{\omega}$ can for the present purpose be partitioned into a vector $\boldsymbol{x}$ of g ($0 < g \leq l$) freely chosen commodities, indexed by the set $G = \{1, ..., g\}$, and a vector $\boldsymbol{y}$ of h ($= l - g$) rationed commodities, indexed by the set $H = \{1, ..., h\}$. Let the vectors $\mathbf{p} \in \mathbf{E}^g_{++}$ and $\mathbf{q} \in \mathbf{E}^h_+$ denote the prices associated with $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively.

The vector $\boldsymbol{z} \in \mathbf{E}_{+}^{h}$ denotes the preallocated quantities of the rationed commodities. The consumer's income, including the value of her endowment, is denoted with Y, where it is assumed that $Y > \mathbf{q} \cdot \boldsymbol{z} \geq 0$ always holds. That is, the outlays on the preallocated commodities do not exhaust the consumer's income.

The $(l+1)$–vector $(\mathbf{p}, \mathbf{q}, Y)$ is called a budget, and the $(l+1+h)$–vector $(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})$ is called a restricted budget. The set of all affordable consumption bundles, the budget set, is the set

$$B(\mathbf{p}, \mathbf{q}, Y) = \{(\mathbf{x}, \mathbf{y}) \in \mathbf{E}_{+}^{l} \;:\; (\mathbf{p}, \mathbf{q}) \cdot (\mathbf{x}, \mathbf{y}) \leq Y\}.$$

Under rationing only a subset of the budget set $B(\mathbf{p}, \mathbf{q}, Y)$ is attainable. This smaller set of feasible consumption bundles, the restricted budget set, is defined as

$$F(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z}) = \{(\mathbf{x}, \mathbf{y}) \in \mathbf{E}_{+}^{l} \;:\; (\mathbf{p}, \mathbf{q}) \cdot (\mathbf{x}, \mathbf{y}) \leq Y,\; \mathbf{y} = \boldsymbol{z}\}.$$

The consumer chooses the most preferred attainable bundle, i.e., the most preferred vector belonging to $F(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})$. The restricted budget set is nonempty, closed and bounded, hence there exists a maximal element in the restricted budget set for the preference preorder. Denote the chosen bundle by $(\mathbf{x}^{*}, \mathbf{y}^{*})$ where $\mathbf{y}^{*} = \boldsymbol{z}$. This bundle is unique for every restricted budget $(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})$.

Rothbarth (1941) pointed out that it is possible to find a vector of artificial or virtual prices, $\boldsymbol{\pi}$, for the rationed goods with the property that if they were charged along with the prices $\mathbf{p}$ for the unrestricted commodities and if given sufficient income, then the consumer would freely chose the consumption bundle $(\mathbf{x}^{*}, \mathbf{y}^{*})$. That is, the bundle $(\mathbf{x}^{*}, \mathbf{y}^{*})$ is the most preferred vector in the budget set $B(\mathbf{p}, \boldsymbol{\pi}, Y_{\pi})$, where $Y_{\pi} = Y + (\boldsymbol{\pi} - \mathbf{q}) \cdot \boldsymbol{z}$ is the virtual income.

The conditions for existence of such a price vector are stated precisely in the following theorem (Bergland 1985). The theorem is a formalizations of results due to Neary and Roberts (1980).

Theorem 7 (Neary-Roberts)
For any restricted budget $(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})$ let the vector $(\mathbf{x}^{*}, \mathbf{y}^{*}) \in F(\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})$ be the most preferred attainable consumption bundle. If the preference relation is a complete preorder that is strongly monotonic, strictly convex from below and smooth, then there exists a vector of strictly positive virtual prices, $\boldsymbol{\pi} \in \mathbf{E}_{++}^{h}$, such that

$$(\mathbf{p}, \boldsymbol{\pi}) \cdot (\mathbf{x}^{*}, \mathbf{y}^{*}) < (\mathbf{p}, \boldsymbol{\pi}) \cdot (\mathbf{x}, \mathbf{y}) \quad \forall\, (\mathbf{x}, \mathbf{y}) \in \mathbf{A}(\mathbf{x}^{*}, \mathbf{y}^{*}) \setminus \left\{(\mathbf{x}^{*}, \mathbf{y}^{*})\right\}.$$

Furthermore, if $\boldsymbol{z} > 0$, then $\boldsymbol{\pi}$ is unique.

The preferences can be represented by a utility function U which is a stricly increasing real-valued continuous concave function defined over $\mathbf{E}^l_+$. In the case of rationing, the consumer's utility maximization problem is expressed by the following nonlinear maximization program:

$$\max_{(x,y)\in\mathbf{E}^l_+} U(x,y) \quad \text{subject to } (p,q)\cdot(x,y) \le Y,\ y = z. \tag{P1}$$

The vector that solves (P1) is the vector of *restricted Marshallian demand functions*, denoted $\left(x^r(p,q,Y,z), y^r(p,q,Y,z)\right)$, where $y^r(\cdot;z) = z$. It follows from the linear budget constraint in (P1) that the restricted demand functions are homogenous of degree zero in budgets.

The value function of the nonlinear maximization program in (P1) is the *restricted indirect utility function,*

$$V^r(p,q,Y;z) = \max_{(x,y)} \{U(x,y) : (x,y) \in F(p,q,Y,z)\}. \tag{13}$$

5.2 The Restricted Income Compensation Function

The dual to the restricted utility maximization program (P1) is the restricted expenditure minimization program defined as:

$$\min_{(x,y)\in\mathbf{E}^l_+} (p,q)\cdot(x,y) \quad \text{subject to} \quad U(x,y) \ge u,\ y = z. \tag{P2}$$

The value function to this program is the restricted expenditure function, e^r, i.e.

$$e^r(p,q,u;z) = \min_{(x,y)} \{(p,q)\cdot(x,y) : U(x,y) \ge u,\ x \ge 0,\ y = z\}. \tag{14}$$

The restricted expenditure function can be rewritten as

$$\begin{aligned} e^r(p,q,u;z) &= \min_x \{(p,q)\cdot(x,y) : U(x,y) \ge u,\ x \ge 0,\ y = z\} \\ &= q\cdot z + \min_x \{p\cdot x : U(x,y) \ge u,\ x \ge 0\} \\ &= q\cdot z + e^c(p,u;z). \end{aligned} \tag{15}$$

The function $e^c(p,u;z)$ the conditional expenditure function, gives the minimum expenditure necessary to obtain utility level u, conditional on the rationing level z.

Let u be the level of satisfaction, or utility, associated with the restricted budget $(\mathbf{p}^0, \mathbf{q}^0, Y^0, \boldsymbol{z}^0)$. The restricted income compensation function is an extension of the income compensation function (Hurwicz and Uzawa 1971). It is defined for the quantity rationing case as:

$$\begin{aligned} \mu^r\big((\mathbf{p}^0, \mathbf{q}^0, \boldsymbol{z}^0); (\mathbf{p}, \mathbf{q}, Y, \boldsymbol{z})\big) &= \min_{\lambda}\{\lambda > 0 : V^r(\mathbf{p}, pbq, \lambda; \boldsymbol{z}) \geq V^r(\mathbf{p}^0, \mathbf{q}^0, Y^0; \boldsymbol{z}^0)\} \\ &= e^r\big(\mathbf{p}, \mathbf{q}, V^r(\mathbf{p}^0, \mathbf{q}^0, Y^0; \boldsymbol{z}^0); \boldsymbol{z}\big). \end{aligned} \tag{16}$$

5.3 Welfare Change Measures

It is possible to define welfare change measures in terms of the gain (or loss) of money income which would measure the gain (or loss) of satisfaction resulting from a change in the restricted budget the consumer faces (Hicks 1943). Let the initial restricted budget situation an individual faces be $(\mathbf{p}^0, \mathbf{q}^0, Y^0, \boldsymbol{z}^0)$, and assume a change resulting in a new restricted budget $(\mathbf{p}^1, \mathbf{q}^1, Y^1, \boldsymbol{z}^1)$. The Hicksian compensating and equivalent welfare change measures are defined by means of the restricted income compensation function as:

Definition 4 *The* compensating *measure,* CM*, is that sum of money received by (if* $CM < 0$*) or from (if* $CM > 0$*) an individual, in order for her to be on the same utility level as before the change;*

$$V^r(\mathbf{p}^1, \mathbf{q}^1, Y^1 - CM; \boldsymbol{z}^1) = V^r(\mathbf{p}^0, \mathbf{q}^0, Y^0; \boldsymbol{z}^0) \tag{17}$$

and hence

$$CM = CM(0, 1) = y^1 - \mu^r\big((\mathbf{p}^1, \mathbf{q}^1, \boldsymbol{z}^1); (\mathbf{p}^0, \mathbf{q}^0, Y^0, \boldsymbol{z}^0)\big). \tag{18}$$

5.4 Probabilistic Choice Formulation

The quantity rationing model of individual behavior provides the behavioral model necessary for linking the described alternatives with the choice data. Let individual t be faced with the choice set C^t which is a particular block of the fractional factorial design randomly assigned to individual t. Let the restricted budget association with alternative $s \in C^t$ be $(\mathbf{p}^s, \mathbf{q}^s, Y^s, \boldsymbol{z}^s)$.

Using the restricted indirect utility function gives the utility level associated with alternative s as:

$$v_s = V^r(\mathbf{p}^s, \mathbf{q}^s, Y^s; \boldsymbol{z}^s).$$

The random utility associated with alternative s is

$$u_s = v_s + \epsilon_s$$

where ϵ_s is some unobserved stochastic term. The choice probability for alternative $c^* \in C^t$ is

$$\begin{aligned} P_{C^t}(c^*) &= \Pr\{u_{c^*} > u_s \quad \forall s \in C^t \setminus \{c^*\}\} \\ &= \Pr\{v_{c^*} + \epsilon_{c^*} > v_s + \epsilon_s \quad \forall s \in C^t \setminus \{c^*\}\}. \end{aligned} \tag{19}$$

Estimation of preference parameters in this model proceeds by using common maximum likelihood techniques for probabilistic choice models (Maddala 1983, Ben-Akiva and Lerman 1985), and expanding the likelihood function to incorporate the structure of the incomplete contingent ranking data from equation 12. Although this is not possible to do directly with currently available statistical packages, there exists a number of general nonlinear optimization routines (Dennis and Schnabel 1983) which can tailored for the current estimation problem. Such routines may also be available through a general econometrics or statistics package.

5.5 Measures of Value

The probabilistic choice modeling framework estimates the restricted indirect utility function. Thus, information is available about the virtual prices for different environmental characteristics through Gorman's Identity, or of the Hicksian welfare change measures. This information can easily be calculated directly from the estimated parameters. In particular can equation 17 be solved for the Hicksian compensating measures.

6 Conclusions

The incomplete ranking method as set forth in this paper offers an alternative technique for determining preferences for complex environmental goods. Contingent valuation with referendum valuation elicitation can be used for this purpose, but the size of the required experiment turns out to be enormous.

In terms of rankings there are both theoretical and empirical evidence supporting the view that the link between choice and ranking breaks down as the rankorder is traversed. Thus a complete ranking of many alternatives may not be the best implementation of ranking experiments, but rather one should use an incomplete ranking with the few best alternatives and, possibly, the worst alternatives. The discussion here indicates that this is a feasible approach, and the estimation of such models are outlined.

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